Circles in General Form

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

Must be equal and have same sign

A and C must be equal and have the same sign in order to be a circle. Other characteristics in this same formula form the equations of an ellipse, a hyperbola, and a parabola.

If you are given an equation in this form, you can work backwards to put the equation in standard form by "completing the square."

Steps to "completing the square"

STEP 1: Rearrange the equation putting the X's together and the Y's together and move the constant to the other side of the equal sign.

STEP 2: Rewrite what you have in step 1, leaving blanks after the x and y terms.

STEP 3: Take ½ of the 2nd x coefficient and square it. Add this number in the blank following the x's. Do the same for the y's. Whatever you add to the left side, remember you must add it to the right side. This preserves equality.

STEP 4: Simplify the right side.

STEP 5: Factor each trinomial on the left and simplify on the right.

$(x \pm \_\_)^2 + (y \pm \_\_)^2 = r^2$

sign is the first sign in the trinomial

Example 1: $x^2 + y^2 + 8x - 2y + 13 = 0$

A and B are equal

$-13 \quad -13$

$x^2 + 8x + \underline{16} + y^2 - 2y + \underline{1} = -13 + 16 + 1$

$\left(\frac{8}{2}\right)^2 = 4^2 = 16 \qquad \left(\frac{2}{2}\right)^2 = 1^2 = 1$

$x^2 + 8x + 16 + y^2 - 2y + 1 = 4$

$(x+4)^2 + (y-1)^2 = 4$

$C = (-4, 1)$

$R = 2$

Example 2: $x^2 + y^2 + 6x - 10y + 9 = 0$

$-9 \quad -9$

$x^2 + 6x + y^2 - 10y = -9$

$x^2 + 6x + \underline{9} + y^2 - 10y + \underline{25} = -9 + 9 + 25$

$\left(\frac{6}{2}\right)^2 = 3^2 = 9 \qquad \left(\frac{-10}{2}\right)^2 = (-5)^2$

$(x^2 + 6x + 9) + (y^2 - 10y + 25) = 25$

$(x+3)^2 + (y-5)^2 = 25$

center $(-3, 5)$

$r = 5$

Example 3: $x^2 + y^2 - 4x + 8y + 4 = 0$

$\qquad\qquad\qquad\qquad\qquad -4 \quad -4$

$x^2 - 4x + y^2 + 8y = -4$

$x^2 - 4x + \underline{4} + y^2 + 8y + \underline{16} = \underbrace{-4 + 4 + 16}$

$\left(\frac{-4}{2}\right)^2 = (-2)^2 \qquad \left(\frac{8}{2}\right)^2 = 4^2$

$(x^2 - 4x + 4) + (y^2 + 8y + 16) = 16$

$(x-2)^2 + (y+4)^2 = 16$

center $(2, -4)$

$r = 4$

Example 4: $x^2 + y^2 + 16x - 12y - 21 = 0$

$\qquad\qquad\qquad\qquad\qquad +21 \quad +21$

$x^2 + 16x + y^2 - 12y = 21$

$x^2 + 16x + \underline{64} + y^2 - 12y + \underline{36} = 21 + 64 + 36$

$\left(\frac{16}{2}\right)^2 = 64 \qquad \left(\frac{-12}{2}\right)^2 = 36$

$(x^2 + 16x + 64) + (y^2 - 12y + 36) = 121$

$(x + 8)^2 + (y - 6)^2 = 121$

Center: $(-8, 6)$ Radius: 11

Example 5: $x^2 + y^2 - 14x + 6y - 6 = 0$

$\qquad\qquad\qquad\qquad\qquad +6 \quad +6$

$x^2 - 14x + y^2 + 6y = 6$

$x^2 - 14x + \underline{49} + y^2 + 6y + \underline{9} = \underbrace{6 + 49 + 9}$

$\left(\frac{-14}{2}\right)^2 = 7^2 \qquad \left(\frac{6}{2}\right)^2 = 3^2$

$(x^2 - 14x + 49) + (y^2 + 6y + 9) = 64$

$(x-7)^2 + (y+3)^2 = 64$

center $(7, -3)$

$r = 8$

Example 6: $\frac{2x^2}{2} + \frac{2y^2}{2} - \frac{8x}{2} + \frac{4y}{2} - \frac{10}{2} = \frac{0}{2}$

How is this one different?

$x^2 + y^2 - 4x + 2y - 5 = 0$

Now Finish up