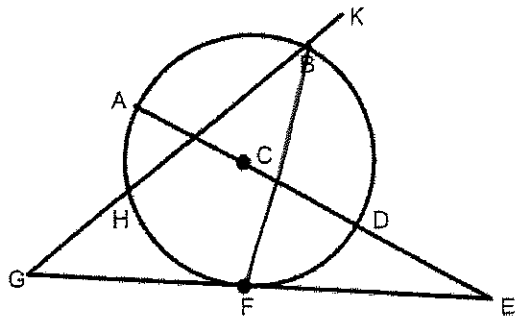
2nd

UNIT 3

Circle Properties

Name: 2nd Period

1: Name the following as a chord, a secant, a tangent, a diameter, or a radius—be specific!



a. $\overline{AD}$ Diameter

b. $\overline{CD}$ radius

c. $\overline{EG}$ Tangent

d. $\overline{HB}$ chord

e. $\overline{FB}$ chord

f. $\overline{FE}$ Tangent

$\overline{KG}$ secant

In the following questions, EF and TU are diameters of Circle M. Find the indicated measure.

2. m$\overset{\frown}{ET}$ 60°

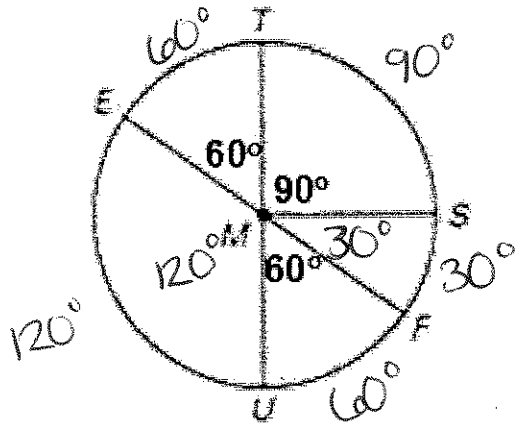
3. m$\overset{\frown}{SF}$ 30°

4. m$\overset{\frown}{ETS}$ 60+90 = 150°

5. m$\overset{\frown}{TSF}$ 90+30 = 120°

6. m$\overset{\frown}{SU}$ 30+60 = 90°

7. m$\overset{\frown}{EU}$ 120°



8. $\overset{\frown}{ESF}$ is a _____ (minor arc, major arc, semicircle)

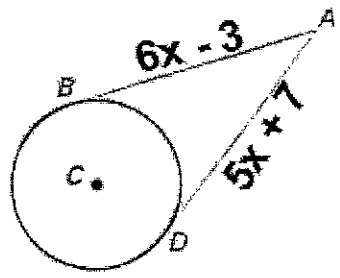
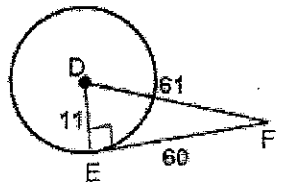
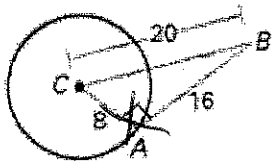
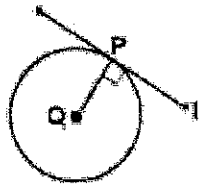
9. $\overset{\frown}{SU}$ is a _____ (minor arc, major arc, semicircle)

10. $\overset{\frown}{ETU}$ is a _____ (minor arc, major arc, semicircle)

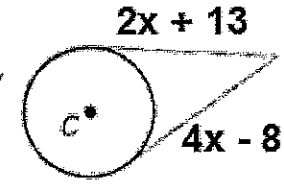
11. $\overset{\frown}{ET}$ is a _____ (minor arc, major arc, semicircle)

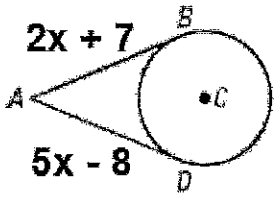
12. $\overset{\frown}{SEU}$ is a _____ (minor arc, major arc, semicircle)

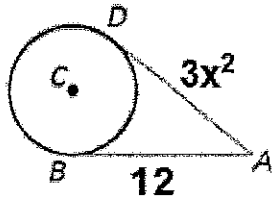
Example 1 – Tangent Properties

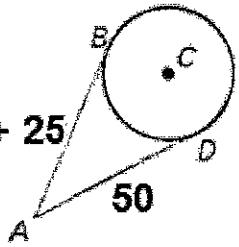
EXAMPLE 1	RULE	WORKED OUT
	2 tangent segments are congruent when they are joined at a common exterior point. Tangent = Tangent	$6x-3=5x+7$ $-5x \quad -5x$ $x-3=7$ $+3 \quad +3$ $x=10$ $\quad \overline{AB}=(6(10))-3$ 57
1. Is $\overline{EF}$ tangent to ⊙D?  **2. Is $\overline{AB}$ tangent to ⊙C?** 	**Tangent Rule:** If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency. If l is tangent to ⊙ Q at P, then $l \perp \overline{QP}$. **Perpendicular Tangent Rule:** In a plane, if a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is tangent to the circle. If $l \perp \overline{QP}$ at P, then l is tangent to ⊙ Q. 	$a^2+b^2=c^2$ (1) $11^2+60^2=c^2$ $3721=c^2$ $61=c$ yes, $\overline{EF}$ is a tangent. (2) $8^2+16^2=c^2$ $320=c^2$ $17.889=c$ $\overline{AB}$ is NOT a tangent.

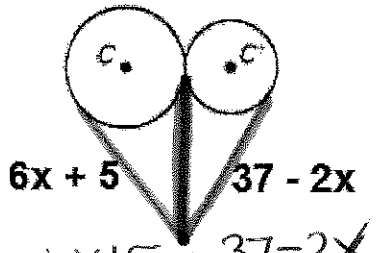
Solve for x using the appropriate property:

3. 

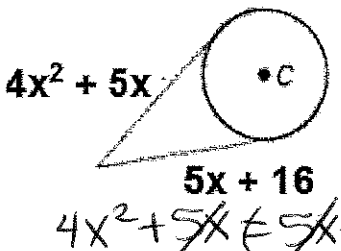
4. 

5. 

6. 

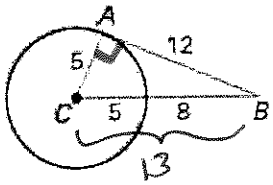
7. 

$6x+5=37-2x$
$+2x \quad +2x$
$8x+5=37$
$-5 \quad -5$
$\frac{8x}{8}=\frac{32}{8}$ $x=4$

8. 

$4x^2+5x=5x+16$
$-5x \quad -5x$
$\frac{4x^2}{4}=\frac{16}{4}$
$x^2=4$
$x=\pm 2$

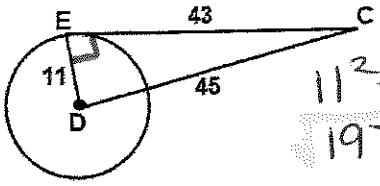
9. Is $\overline{AB}$ tangent to $\odot C$?



$5^2+12^2=c^2$
$169=c^2$
$13=c$

yes, $\overline{AB}$ is a tangent

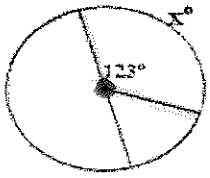
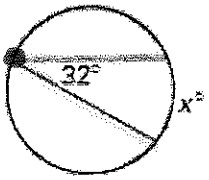
10. Is $\overline{CE}$ tangent to $\odot D$?



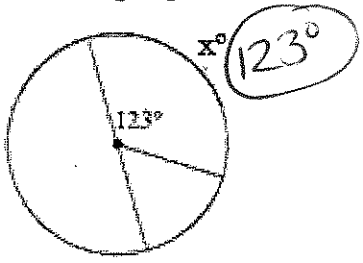
$11^2+43^2=c^2$
$1970=c^2$
$44.385=c$

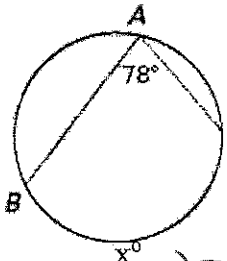
$\overline{CE}$ is Not a tangent.

Example 2 – Central and Inscribed Angles

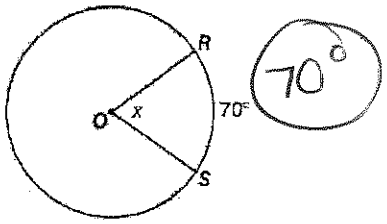
EXAMPLE 2	RULE	WORKED OUT
	A central angle is equal to the intercepted arc. Central Angle = Arc central same	$x=123^\circ$
	An inscribed angle is ½ the intercepted arc. Arc = 2(Angle) Angle = $\frac{\text{Arc}}{2}$	$x=2(32)$ $=64^\circ$

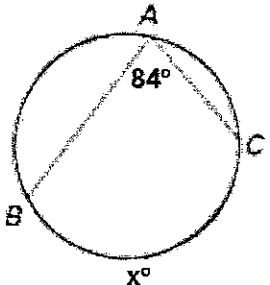
Find the missing angle:

1. 

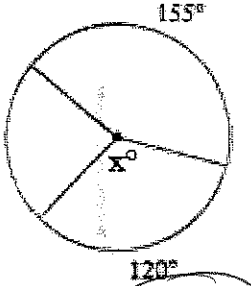
2. 

$2(78)=156^\circ$

3. 

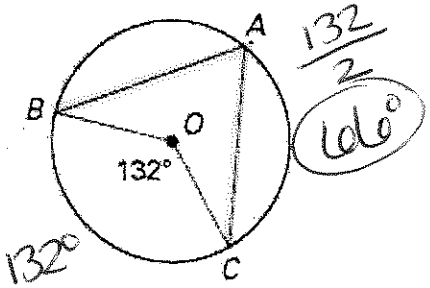
4. 

$2(84)=168^\circ$

5. 

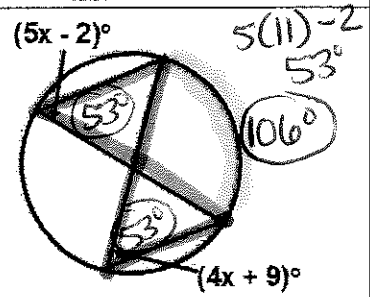
120°

6 $m\angle BAC =$?



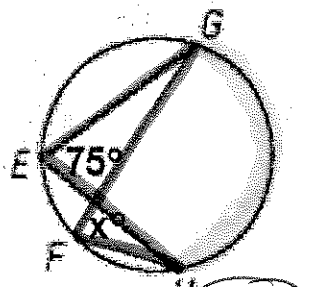
$\frac{132}{2}=66^\circ$

Example 3 – Inscribed Angles that Share an Intercepted Arc

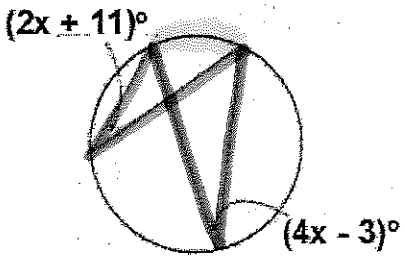
EXAMPLE 3	RULE	WORKED OUT
	If inscribed angles intercept the same arc, they are congruent. Angle = Angle	$5x-2=4x+9$ $-4x \quad -4x$ $x-2=9$ $+2 \quad +2$ $x=11$

Solve for x or find the angle requested.

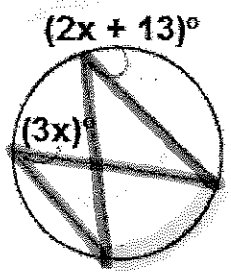
1.



$x = 75°$

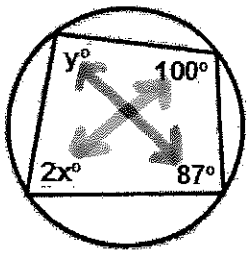


$2x+11=4x-3$
$-2x \quad -2x$
$11=2x-3$
$+3 \quad +3$
$\frac{14}{2}=\frac{2x}{2}$ $x=7$



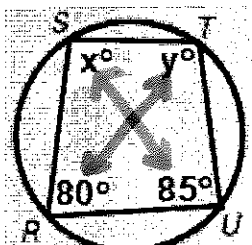
$3x=2x+13$
$-2x \quad -2x$
$x=13$

Example 4 – Inscribed Quadrilaterals

EXAMPLE 4	RULE	WORKED OUT
	When a quadrilateral is inscribed in a circle, opposite angles are supplementary (add up to 180). Opp Angle + Opp Angle = 180	$2x+100=180$ $-100 \quad -100$ $\frac{2x}{2}=\frac{80}{2}$ $x=40$ $y+87=180$ $-87 \quad -87$ $y=93$

Solve for the missing variables.

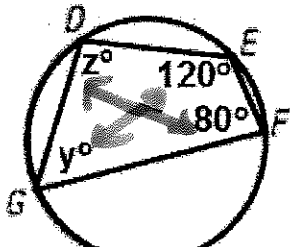
1.



$x+85=180$
$-85 \quad -85$
$x=95$

$y+80=180$
$-80 \quad -80$
$y=100$

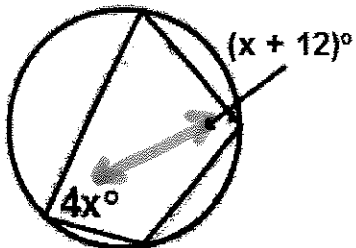
2.



$y+120=180$
$-120 \quad -120$
$y=60$

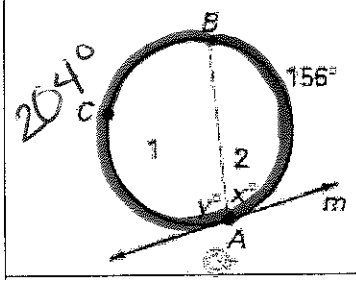
$z+80=180$
$-80 \quad -80$
$z=100$

3.

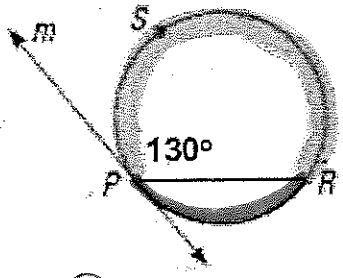


$4x+x+12=180$
$5x+12=180$
$-12 \quad -12$
$\frac{5x}{5}=\frac{168}{5}$
$x=33.6$

Example 5 – Intersecting Chords and Tangents

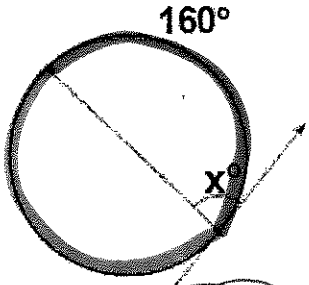
EXAMPLE 5	RULE	WORKED OUT
	If a chord and a tangent intersect on the circle, the measure of the angle is ½ the measure of the intercepted arc. Arc = 2(Angle) Angle = Arc / 2	$x = \frac{156}{2}$ $y = \frac{204}{2}$ $x = 78$ $y = 102$

1. Find the m $\overset{\frown}{PSR}$



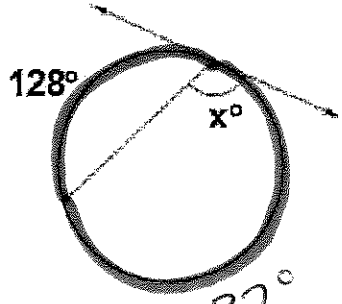
130 × 2 = 260°

2. Solve for x.



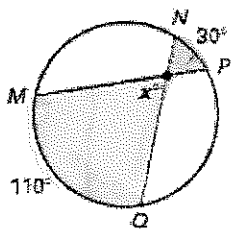
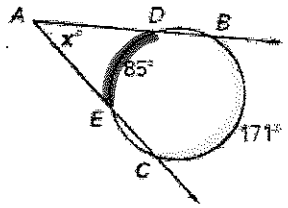
$\frac{160}{2} = 80°$

3. What is $m\angle x$?



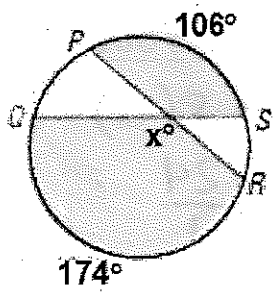
232° $\frac{232}{2} = 116°$

Example 6 – Interior and Exterior Angles

EXAMPLE 6	RULE	WORKED OUT
	If 2 chords intersect inside a circle, then the measure of each angle is ½ the sum of the measures of the arcs intercepted by the angle and its vertical angle. Arc + Arc / 2 = Inside Angle	$\frac{110 + 30}{2}$ 70°
	If a tangent and a secant, 2 tangents, or 2 secants intersect in the exterior of a circle, the measure of the angle formed is ½ the difference of the measures of the intercepted arcs. Big Arc – Little Arc / 2 = Ext Angle	$\frac{171 - 85}{2}$ 43°

Solve for x:

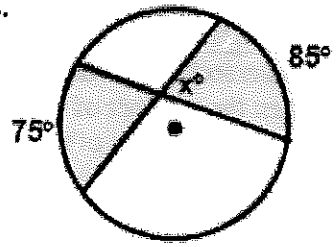
1.



$\frac{174+106}{2}$

$140°$

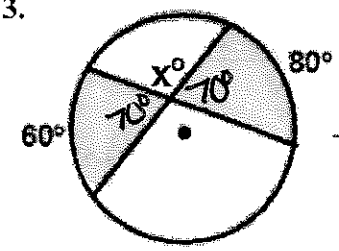
2.



$\frac{75+85}{2}$

$80°$

3.



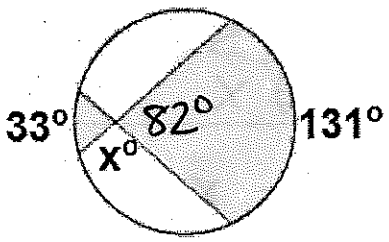
$\frac{60+80}{2}$

$70°$

$180-70$

$110°$

4.

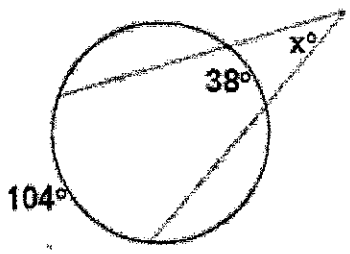


$\frac{33+131}{2}$

$82°$

$180-82=98°$

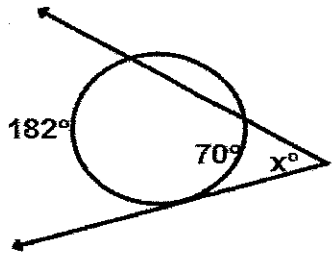
5.



$\frac{104-38}{2}$

$33°$

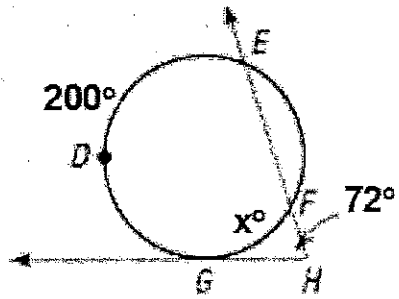
6.



$\frac{182-70}{2}$

$56°$

7.



$\frac{200-x}{2}=72(2)$

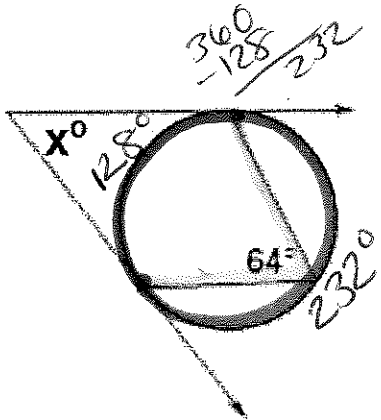
$200-x=144$

$-200 \quad -200$

$\frac{-x}{-1}=\frac{-56}{-1}$

$x=56°$

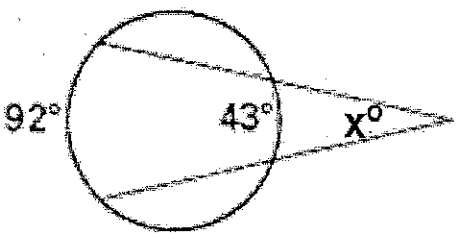
8.



$\frac{232-128}{2}$

$x=52°$

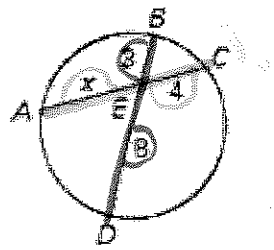
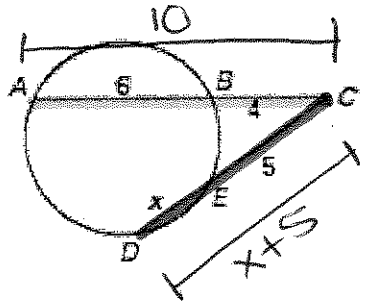
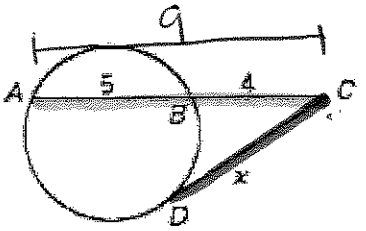
9.



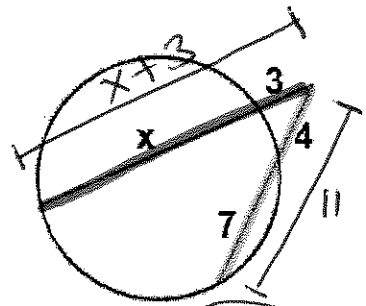
$\frac{92-43}{2}$

$24.5°$

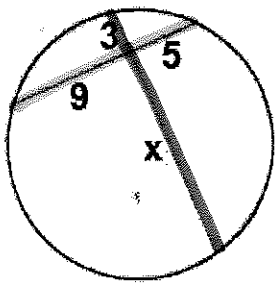
Example 7 – Segments formed by chords, secants, and tangents

EXAMPLE 7	RULE	WORKED OUT
	If 2 chords intersect in the interior of a circle then the product of each chord is congruent to the other. Chord 1 Chord 2 Part · Part = Part · Part	$x(4) = 8(3)$ $\frac{4x}{4} = \frac{24}{4}$ $x = 6$
	2 secant segments share the same exterior endpoint, then the product of the length of 1 secant segment and the length of its external segment = the product of the length of the other secant segment and the length of its external segment. Secant 1 Secant 2 Outside(whole) = Outside(whole)	$4(10) = 5(x+5)$ $40 = 5x + 25$ $-25 \quad -25$ $\frac{15}{5} = \frac{5x}{5}$ $3 = x$
	A secant segment and a tangent segment share an exterior endpoint, then the product of the length of the secant segment & its external segment equals the square of the tangent segment length. Secant Tangent Outside(whole) = Outside(Outside) outside (whole)	$4(9) = x(x)$ $\sqrt{36} = \sqrt{x^2}$ $6 = x$

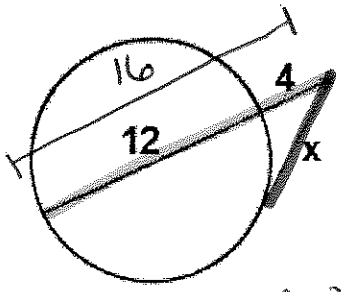
Solve for x.

1. 

$4(11) = 3(x+3)$
$44 = 3x + 9$
$-9 \quad -9$
$\frac{35}{3} = \frac{3x}{3}$
$11.667 = x$

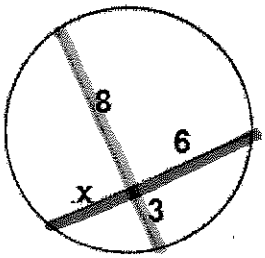
2. 

$9(5) = 3(x)$
$\frac{45}{3} = \frac{3x}{3}$
$15 = x$

3. 

$4(16) = x(x)$
$\sqrt{64} = \sqrt{x^2}$
$8 = x$

4.

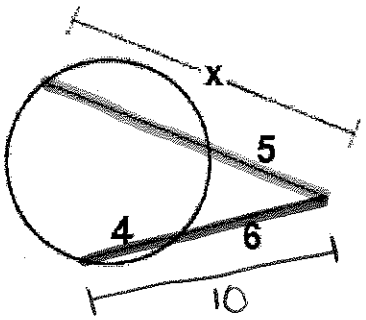


$3(8) = x(6)$

$\frac{24}{6} = \frac{6x}{6}$

$4 = x$

5

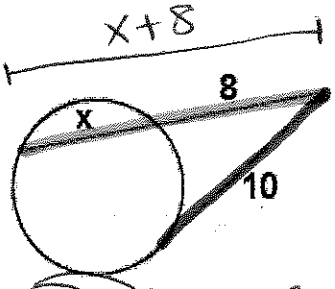


$5(x) = 6(10)$

$\frac{5x}{5} = \frac{60}{5}$

$x = 12$

6.



$8(x+8) = 10(10)$

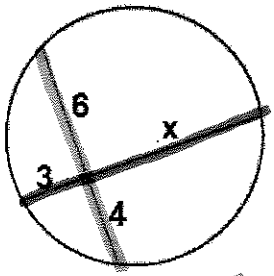
$8x + 64 = 100$

$-64 \quad -64$

$\frac{8x}{8} = \frac{36}{8}$

$4.5 = x$

7.

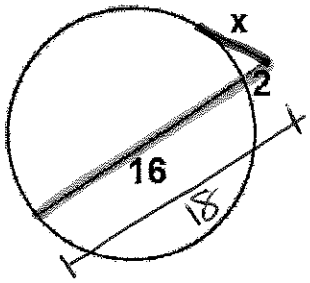


$6(4) = 3(x)$

$\frac{24}{3} = \frac{3x}{3}$

$8 = x$

8.

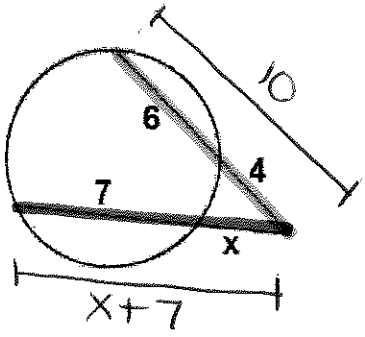


$2(18) = x(x)$

$\sqrt{36} = \sqrt{x^2}$

$6 = x$

9.



$4(10) = x(x+7)$

$40 = x^2 + 7x$

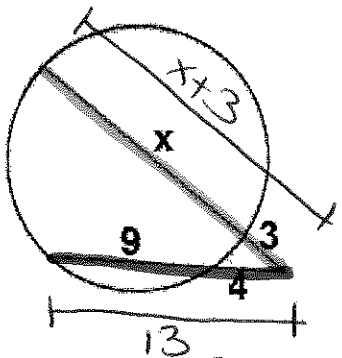
$-40 \quad -40$

$0 = x^2 + 7x - 40$

$x = \frac{-7 \pm \sqrt{(7)^2 - 4(1)(-40)}}{2(1)}$

$\frac{-7 \pm \sqrt{209}}{2}$ → 3.728, −10.728

10.



$4(13) = 3(x+3)$

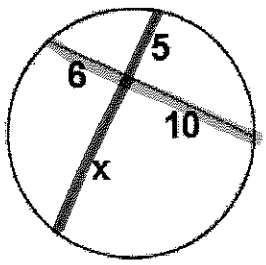
$52 = 3x + 9$

$-9 \quad -9$

$\frac{43}{3} = \frac{3x}{3}$

$14.333 = x$

11.

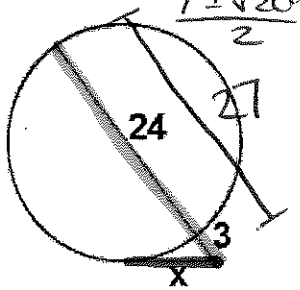


$6(10) = 5(x)$

$\frac{60}{5} = \frac{5x}{5}$

$12 = x$

12.

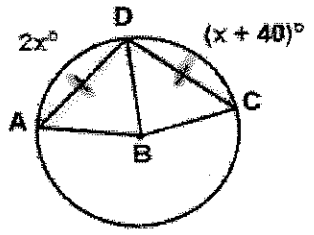
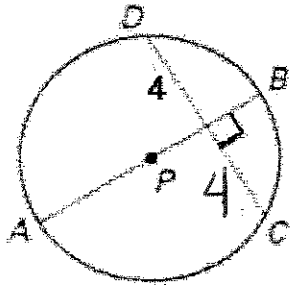
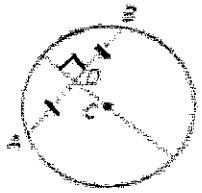
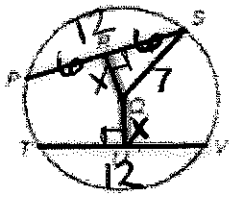


$3(27) = x(x)$

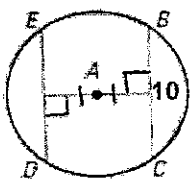
$\sqrt{81} = \sqrt{x^2}$

$9 = x$

Example 8 – Chord Properties

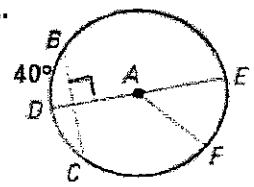
EXAMPLE 8	RULE	WORKED OUT
Find $m\overset{\frown}{AD}$. 	**Congruent Chord and Arc** In the same circle, or in congruent circles, 2 minor arcs are congruent if and only if their corresponding chords are congruent.	$2x = x + 40$ $-x \quad -x$ $x = 40$ $m\overset{\frown}{AD} = 2x = 2(40) = 80°$
EX. 1: DC = _____ 	**Diameters and Chords** * If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc. * If one chord is a perpendicular bisector of another chord, then the first chord is a diameter. 	DC = 8
EX. 1: PS = 12 TV = 12 SQ = 7 Find QU. 	**Congruent Chords** In the same circle, or in congruent circles, 2 chords are congruent if and only if they are equidistant from the center.	$a^2 + b^2 = c^2$ $x^2 + 6^2 = 7^2$ $x^2 + 36 = 49$ $-36 \quad -36$ $\sqrt{x^2} = \sqrt{13}$ $x = 3.606$

1.



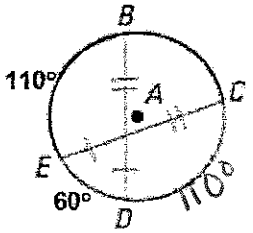
$m\overline{ED} = 10$

2.



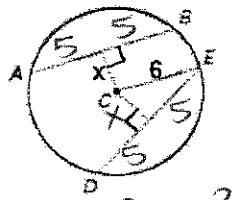
$m\overset{\frown}{DC} = 40°$

3.



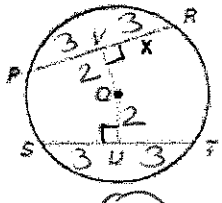
$m\overset{\frown}{EDC} = 170°$

4. AB = DE = 10
radius = 6
Find x.



$x^2 + 5^2 = 6^2$
$x^2 + 25 = 36$
$-25 \quad -25$
$\sqrt{x^2} = \sqrt{11}$
$x = 3.317$

5. QV = 2
QU = 2
SU = 3
Find x.



$x = 3$