# UNIT 5 

## QUADRATIC FUNCTIONS

Packet 4

## GRAPHING AND

## REAL WORLD APPLICATION

Notes and Practice - Characteristics of a Graph
Notes and Practice - Real World Application
Notes and Practice - Odd and Even Functions
Notes and Practice - Scatter Plots

## Overview of Characteristics:

## Direction of the graph:

- Use the "a" value. If "a" is positive the graph will open up. If "a" is negative the graph with open down.

Vertex (minimum / maximum / extrema)

- Standard Form Step 1 - Find the x -coordinate by using $\mathrm{x}=\frac{-b}{2 a}$

Step 2 - Find the $y$-coordinate by substituting into the original function.

- Vertex Form $f(x)=a(x-h)^{2}+k$ The Vertex is $(h, k)$


## Axis of Symmetry (AOS)

- $x=x$-value of the vertex. For example. If the vertex is $(3,4)$, the AOS will be $x=3$.


## y-intercept

- Substitute 0 in for x and solve for y . This will give you the y -intercept.


## x-intercepts

- When in standard form you can determine the x-intercepts using a variety of methods:
- Isolating $x$ (if possible)
- Factoring - set each factor equal to 0 and solve for x .
- Quadratic Formula


## Domain

- These are the $x$-values (input values) of your function. How far to the left and right can your graph go?
- For a quadratic function the domain will be $(-\infty, \infty)$ unless there is a restricted domain.


## Range

- These are the $y$-values (output values) of your function. How far up or down can your graph go?
- For a quadratic function the $y$-value of the vertex will be included in the range as the vertex is the minimum or maximum on the graph depending on the direction of the graph.


## Graphing Quadratic Functions

There are 3 different forms you need to be familiar with:

1. Standard Form

$$
f(x)=x^{2}-2 x-8
$$

2. Vertex Form

$$
f(x)=(x-1)^{2}-9
$$

3. Intercept Form

$$
f(x)=(x+2)(x-4)
$$

Each of these 3 functions is the exact same function... just in a different form.

Vertex $(1,-9)$
A.O.S. $\quad x=1$
$x$-int. $\quad(-2,0)$ and $(4,0)$
$y$-int. $\quad(0,-8)$


Do you see any of these values in the equations above?? What are the benefits of each equation?

## Benefits of Quadratic Functions in Different Forms:

| Standard Form | Vertex Form | Intercept Form |
| :---: | :---: | :---: |
| $y=x^{2}-2 x-8$ | $y=(x-1)^{2}-9$ | $y=(x+2)(x-4)$ |
|  |  |  |

Example 1. $f(x)=-8+2 x+x^{2}$

Standard Form: $\qquad$
$\mathrm{a}=$ $\qquad$ b $=$ $\qquad$ $\mathrm{c}=$

Does it open up or down? $\qquad$
Vertex: $\qquad$
Axis of Symmetry: $\qquad$
y-intercept: $\qquad$
x-intercept(s): $\qquad$


Domain: $\qquad$ Range: $\qquad$


Example 2: $f(x)=(x+3)^{2}-4$
Does it open up or down? $\qquad$
Vertex: $\qquad$
Axis of Symmetry: $\qquad$
y-intercept: $\qquad$
x-intercept(s): $\qquad$
Domain: $\qquad$
Range: $\qquad$



Example 3: $f(x)=12 x-9-3 x^{2}$
Standard Form: $\qquad$
$\mathrm{a}=$ $\qquad$ $b=$ $\qquad$ $\mathrm{C}=$

Does it open up or down? $\qquad$
Vertex: $\qquad$
Axis of Symmetry: $\qquad$
y-intercept: $\qquad$
x-intercept(s): $\qquad$



Domain: $\qquad$ Range: $\qquad$
4. $f(x)=x^{2}+7-8 x$

Standard Form: $\qquad$
$\mathrm{a}=$ $\qquad$ b = $\qquad$ $\mathrm{c}=$ $\qquad$

Does it open up or down? $\qquad$
Vertex: $\qquad$
Axis of Symmetry: $\qquad$
y-intercept: $\qquad$
x-intercept(s): $\qquad$

| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| :--- | :--- |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

Domain: $\qquad$ Range: $\qquad$
5. $f(x)=-2(x-3)(x+1)$

Standard Form: $\qquad$
$\mathrm{a}=$ $\qquad$ $b=$ $\qquad$ $\mathrm{c}=$

Does it open up or down? $\qquad$
Vertex: $\qquad$
Axis of Symmetry: $\qquad$
y-intercept: $\qquad$
x-intercept(s): $\qquad$

| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| :--- | :--- |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

Domain: $\qquad$ Range: $\qquad$

6. $f(x)=-2(x+3)^{2}+8$

Does it open up or down? $\qquad$
Vertex: $\qquad$
Axis of Symmetry: $\qquad$
$y$-intercept: $\qquad$
x-intercept(s): $\qquad$
Domain: $\qquad$
Range: $\qquad$

| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| :---: | :---: |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |



## Quadratics in Real Life

In most cases, $\mathrm{x}=$ time and $\mathrm{y}=$ height

Example 1: Johnny throws a tennis ball as far as he can. We have the following function $f(x)=-4 x^{2}+8 x+7$ expressing a function of $t$ where $x$ represents the number of seconds since the tennis ball left Johnny's hand and $f(x)$ represents the height of the tennis ball in feet.
a. Graph the function.
b. Identify the $y$-intercept.
c. Identify the vertex.
d. The graph should not go outside of the first quadrant. Why?
e. What do the $x$ and $y$ axis represent?
f. What is the maximum height of the tennis ball?
g. How many seconds does it take for the ball to reach its maximum height?

h. How many seconds does it take for the ball to hit the ground?
i. After 1.5 seconds has past, is the ball going up or down?
j. After 0.5 seconds has past is the ball going up or down?
k. After 1 second has past, is the ball going up or down?
I. What interval is the ball going up?
m. What interval is the ball going down?

Example 2: A soccer ball is kicked by a super hero. We have the following function $f(x)=-16 t^{2}+48 t$ where $t$ represents the number of seconds since the soccer ball left the ground and $f(t)$ represents the height of the ball in feet.
a. Graph the function.
b. Identify the $y$-intercept.
c. Identify the vertex.
d. What do the $x$ and $y$ axis represent?
e. What is the maximum height of the soccer ball?
f. How many seconds does it take for the ball to reach its maximum height?

g. How many seconds does it take for the ball to hit the ground?
h. After 2 seconds has past, is the ball going up or down?
i. How high is the ball at exactly 2 seconds?
j. What interval is the ball going up?
k. What interval is the ball going down?

## Complete problems 1-5. You MUST draw a sketch with every question that accurately represents the problem.

1. Bobby hits a golf ball. The following function models the height, $f(x)$, in feet, of an object $x$ seconds after it is hit in the air: $f(x)=-16 x^{2}+96 x$.
a. What time is the golf ball highest in the air?
b. How high is it at it highest point?

2. Jason Heyward hits a baseball. The following function models the height, $h(t)$, in feet, of an object $t$ seconds after it is hit in the air: $f(t)=-16 t^{2}+64 t+3$.
a. What is the highest the baseball will go?
b. Is the ball going up or down after 2.5 seconds?

3. A shoe is thrown off a cliff. The following function models the height, $f(x)$, in feet, of an object $x$ seconds after it is in the air: $f(x)=-16 x^{2}+24 x+150$.
a. Is the shoe going up, down, or at its highest point after exactly 3 seconds?
b. How high is it after exactly 3 seconds?

4. Bozo the clown is shot out of a human canon at the circus. The following function models the height, $h(t)$, in feet, of an object $t$ seconds after it is in the air: $f(t)=-16 t^{2}+32 t+5$.
a. Is Bozo going up or down after exactly 2 seconds?
b. How high off the ground is he after exactly 2 seconds?

5. A nerf gun shoots a foam dart into the air. The following function models the height, $f(x)$, in feet, of an object $x$ seconds after it is in the air: $f(x)=-16 x^{2}+16 x+6$.
a. Is the foam dart going up or down after 1 second?
b. How high is the foam dart at 2 seconds?

Can this happen?

6. A ball is thrown into the air from a height of 4 feet at time $t=0$. The function that models this situation is $h(t)=-16 t^{2}+63 t+4$, where $t$ is measured in seconds and $h$ is the height in feet.
a. What is the height of the ball after 2 seconds?
b. When will the ball reach a height of 50 feet?
c. What is the maximum height of the ball?
d. When will the ball hit the ground?
e. What domain makes sense for the function?
7. The function $s(t)=v t+h-0.5$ at 2 represents the height of an object, $s$, from the ground after time, $t$, when the object is thrown with an initial velocity of $v$, at an initial height of $h$, and where $a$ is the acceleration due to gravity ( 32 feet per second squared). A baseball player hits a baseball 4 feet above the ground with an initial velocity of 80 feet per second. About how long will it take the baseball to hit the ground?
A. 2 seconds
B. 3 seconds
C. 4 seconds
D. 5 seconds
8. A flying disk is thrown into the air from a height of 25 feet at time $t=0$. The function that models this situation is $h(t)=-16 t^{2}+75 t+25$, where $t$ is measured in seconds and $h$ is the height in feet. What values of $t$ best describe the times when the disk is flying in the air?
A. $0<t<5$
B. $0<t<25$
C. all real numbers
D. all positive integers
9. An object is thrown in the air with an initial velocity of $5 \mathrm{~m} / \mathrm{s}$ from a height of 9 m . The equation $h(t)=-4.9 t^{2}+5 t+9$ models the height of the object in meters after $t$ seconds. How many seconds does it take for the object to hit the ground?
A. 0.94 seconds
B. 1.77 seconds
C. 1.96 seconds
D. 9.0 seconds

## Even, Odd, or Neither Functions

There are 2 ways to tell if a function is odd or even... Algebraically and Graphically.

## Algebraically:

A function is $\qquad$ if all variable exponents are even.

A function is $\qquad$ if all variable exponents are odd.

BEWARE OF CONSTANTS
All constants
really have a $x^{0}$

A function is $\qquad$ if the exponents are a mixture of odd and even.

## Examples:

1. $f(x)=x^{3}-x$
2. $f(x)=x^{2}+1$
3. $f(x)=x^{2}+6 x-4$
4. $f(x)=2 x^{4}-3$
5. $f(x)=-x^{3}$
6. $f(x)=x^{2}+4$

## Graphically:

A function is $\qquad$ if the graph reflects across the $y$-axis.
(meaning... you can fold it hotdog style and it would match up with the y-axis in the middle)

A function is $\qquad$ if the graph has $180^{\circ}$ rotational symmetry about the origin. (meaning... you can turn it upside down and it will look exactly the same... it also must go through the origin).

## Examples:

1. 


2.

3.

4.

5.

6.


## You Practice:

Determine whether the following functions are even, odd, or neither.
1.

4.

7. $f(x)=3 x^{2}$
8. $f(x)=x^{3}-2$
9. $f(x)=3 x+4$
10. $f(x)=x^{2}-5$
11. $f(x)=10 x+5$
12. $f(x)=2(x+1)^{2} \quad$ Hint: Multiply $1^{\text {st }}$

## Scatter Plots:

What type of graph would most likely fit the scatter plot shown? (Linear, Quadratic, or Exponential)







Given the Tables below, what type of graph would it form?

| X | Y | X | Y | X | $Y$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 1 | 3 | 1 | 3 |
| 2 | 4 | 2 | 6 | 2 | 4 |
| 3 | 8 | 3 | 11 | 3 | 5 |
| 4 | 16 | 4 | 18 | 4 | 6 |
| 5 | 32 | 5 | 27 | 5 | 7 |
| 6 | 64 | 6 | 38 | 6 | 8 |

$f(x)=3+4 x+x^{2}$

## Standard Form:

$a=$ $\qquad$ $b=$ $\qquad$ $\mathrm{c}=$ $\qquad$

Does it open up or down? $\qquad$
$y$-int: $\qquad$
$x-i n t(x):$ $\qquad$

Vertex: $\qquad$

AOS: $\qquad$

Domain: $\qquad$

Range: $\qquad$


