NAME: ______________________________ PERIOD: _______

Algebra II
UNIT 1

Quadratics Revisited

Algebra II - Unit 1: Revisiting Quadratics

WHAT ARE YOU LEARNING?

Henry County Graduate Learner Outcomes:

- As a Henry County graduate, I will be able to create, interpret, use, and analyze patterns of algebraic structures to make sense of problems.
- As a Henry County graduate, I will be able to use functions to interpret and analyze a variety of contexts.

Georgia Standards of Excellence:

Lesson 1-1 – Simplifying Exponents

MGSE9-12.N.RN.1 Explain how the meaning of rational exponents follows from extending the properties of integer exponents to rational numbers, allowing for a notation for radicals in terms of rational exponents. For example, we define 5(1/3) to be the cube root of 5 because we want $[5(1/3)]^3 = 5[(1/3) \times 3]$ to hold, so $[5(1/3)]^3$ must equal 5.

Lesson 1-2 – Rewriting between Rational and Radical Notation and Simplifying Radicals

MGSE9-12.N.RN.2 - Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Lesson 1-3 – Complex Operations and Equations

MGSE9-12.N.CN.1 - Understand there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi where a and b are real numbers.

MGSE9-12.N.CN.2 - Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

MGSE9-12.N.CN.3 - Find the conjugate of a complex number; use the conjugate to find the quotient of complex numbers.

Lesson 1-4 – Solving Quadratics by Factoring

MGSE9-12.N.CN.7 - Solve quadratic equations with real coefficients that have complex solutions by (but not limited to) square roots, completing the square, and the quadratic formula.

MGSE9-12.N.CN.8 - Extend polynomial identities to include factoring with complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$.

Lesson 1-5 – The Discriminant and the Quadratic Formula

MGSE9-12.A.REI.4 - Solve quadratic equations in one variable.

MGSE9-12.A.REI.4b - Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, factoring, completing the square, and the quadratic formula, as appropriate to the initial form of the equation.

Algebra II - Unit 1: Revisiting Quadratics

WHY ARE YOU LEARNING THIS?

Level 3 Performance Task:

Sorting Task – Solving quadratic equations by different methods.

Since solving is such a huge part of this unit you will be asked to sort various types of equations into four categories. In doing so you will determine the best method for solving each type of equation. Once sorting is completed, each equation will be solved using the identified method.

WHAT IS YOUR GOAL FOR THIS UNIT?

Unit Goal:

I scored a _________ on my pretest.

My goal is to score a _________ or higher on the end of unit test.

To achieve this goal I will ____

____

HOW WILL YOU KNOW WHEN YOU'VE MASTERED THIS? SHOW ME THE EVIDENCE!

Data Analysis: **Pre-Test Score** ______________ **Post-Test Score** ______________

Learning Targets:	Pre-Test Score	Quiz Score	Post-Test Score
K1: I can extend the properties of integer exponents to rational numbers. (N.RN.1)			
R1: I can rewrite expressions involving radical and rational exponents. (N.RN.2)			
K2: I can simplify a complex number "i." For example, $i^2 = -1$. (N.CN.1)			
K3: I can add, subtract, and multiply complex numbers. (N.CN.2)			
R2: I can find a conjugate and use the conjugate to find the quotient of complex numbers. (N.CN.3)			
R3: I can solve quadratic equations to include complex solutions. (A.REI.4b, N.CN.7)			
R4: I can factor polynomials to include complex factors. (N.CN.8)			

LEARNING ACTIVITIES

Lesson 1-1 – Simplifying Exponents

K1: I can extend the properties of integer exponents to rational numbers. (N.RN.1)

_____ Watch video lesson and take notes
_____ Complete guided notes with teacher
_____ Complete 1-1 practice

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Lesson 1-2 – Rewriting between Rational and Radical Notation and Simplifying Radicals

R1: I can rewrite expressions involving radical and rational exponents. (N.RN.2)

_____ Watch video lesson and take notes
_____ Complete g:uided notes with teacher
_____ Complete 1-2 practice

Quiz 1-1 to 1-2

Lesson 1-3 – Complex Operations and Equations

K2: I can simplify a complex number "i." For example, $i^2 = -1$. (N.CN.1)

_____ Watch video lesson and take notes
_____ Complete guided notes with teacher
_____ Complete 1-3 practice

K3: I can add, subtract, and multiply complex numbers. (N.CN.2)

_____ Watch video lesson and take notes
_____ Complete guided notes with teacher
_____ Complete 1-3 practice

R2: I can find a conjugate and use the conjugate to find the quotient of complex numbers. (N.CN.3)

_____ Watch video lesson and take notes
_____ Complete guided notes with teacher
_____ Complete 1-3 practice

Quiz 1-3

Lesson 1-4 – Solving Quadratics by Factoring

R3: I can solve quadratic equations to included complex solutions. (A.REI.4b, N.CN.7)

_____ Watch video lesson and take notes
_____ Complete guided notes with teacher
_____ Complete 1-4 practice

R4: I can factor polynomials to include complex factors. (N.CN.8)

_____ Watch video lesson and take notes
_____ Complete guided notes with teacher
_____ Complete 1-4 practice

Lesson 1-5 – The Discriminant and the Quadratic Formula

R3: I can solve quadratic equations to included complex solutions. (A.REI.4b, N.CN.7)

_____ Watch video lesson and take notes
_____ Complete guided notes with teacher
_____ Complete 1-5 practice

UNIT 1 Assessments

_____ Complete Unit 1 Review Guide
_____ Complete Unit 1 Performance Task
_____ Complete Unit 1 Test
_____ Complete Unit 1 Reflection

Algebra II - Unit 1: Revisiting Quadratics

Lesson 1-1: Simplifying Exponents

Standard: MGSE9-12.N.RN.1 Explain how the meaning of rational exponents follows from extending the properties of integer exponents to rational numbers, allowing for a notation for radicals in terms of rational exponents.

Learning Target: K1: ____

____

Properties of Exponents

Property	Rule	Helpful Hint	Example
Product of Powers	$a^m \bullet a^n = a^{m+n}$	$\times \rightarrow +$	$x^3 \cdot x^2 =$ $x^{2/3} \cdot x^{1/5} =$
Power of a Power	$(a^m)^n = a^{m \bullet n}$	$\text{Power}^{\text{Power}} \rightarrow \times$	$(x^3)^2 =$ $(x^{3/4})^{2/3} =$
Power of a Product	$(ab)^m = a^m b^m$	Distribute the exponent	$(3x)^4 =$
Negative Exponent	$a^{-m} = \frac{1}{a^m}$, $a \neq 0$	Turn that frown upside down!	$3^{-2} =$ $\frac{y^2x^{-3}z^5}{a^{-5}b^2c^{-3}} =$
Zero Exponent	$a^0 = 1$, $a \neq 0$	Zero Power cannot be zero	$4^0 =$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}$, $a \neq 0$	$\div \rightarrow -$	$\frac{x^7}{x^4} =$ $\frac{x^{2/3}}{x^{1/3}} =$
Power of a Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, $b \neq 0$	Distribute the exponent	$\left(\frac{5}{3}\right)^2 =$ $\left(\frac{2}{3}\right)^{-3} =$

Algebra II - Unit 1: Revisiting Quadratics

More Examples and Practice – Simplify the following:

1.) $\left(x^{2/5}\right)^{3/4}$

2.) $\dfrac{y^2x^{-3}z^5}{y^{-4}x^6z^7}$

3.) $\dfrac{y^6}{x^9 \cdot z^2}$

4.) $x^{\frac{1}{5}} \cdot x^{\frac{3}{5}}$

5.) $\dfrac{a^{\frac{5}{3}}}{a^{\frac{1}{3}}}$

6.) $\dfrac{m^{\frac{2}{3}} \cdot m^{-\frac{1}{2}}}{m^{\frac{1}{6}}}$

7.) $\left(b^{\frac{1}{4}}\right)^4$

8.) $\left(p^{\frac{3}{4}}\right)^{\frac{2}{5}}$

9.) $\dfrac{w^{\frac{1}{6}}}{w^{-\frac{2}{3}}}$

10.) $7^4 \cdot 7^{12}$

11.) $t^{-4} \cdot t^{15}$

12.) $3^{2/3} \cdot 3^{3/4}$

13.) $(-8)^8 \cdot (-8)^{-13}$

14.) $\dfrac{b^8}{b^6}$

15.) $\dfrac{b^{1/3}}{b^{1/4}}$

16.) $\dfrac{11^{14}}{11^{18}}$

17.) $(9^5)^4$

18.) $5^{1/5} \cdot 5^{2/3}$

19.) $(c^2)^9$

20.) $\left(\dfrac{5}{3}\right)^4$

21.) $(6^{3/4})^{4/5}$

22.) $\left(\dfrac{7}{2}\right)^{-3}$

22.) $\dfrac{h^4x^{-7}}{p^{-5}b^{18}}$

23.) $\left(\dfrac{4}{5}\right)^{1/2}$

24.) $\frac{10a^4b^7}{12a^8b^5}$

25.) $(3x^4)(2x^3)$

26.) $\left(5p^4q^2\right)^2$

Challenge Questions:

27.) $\left(-3m^3n\right)^2\left(2m^2n^4\right)$

28.) $\frac{18\text{ a}^{-3}\text{b}^2\text{c}^6}{24\text{ a}^2\text{b}^5\text{c}^4}$

29.) $\left(\frac{2\text{x}^5\text{y}^3\text{z}^{-2}}{5\text{x}^2\text{y}^4}\right)^2$

30.) $\left(6a^2b^5c\right)\left(4a^3b^{-2}c^8\right)$

31.) $\left(4p^3q^{-2}t^4\right)^2$

32.) $\left(\frac{4x^2y^{-3}}{y^{-2}}\right)^{-1}$

33.) $\left(8x^7y^3\right)\left(3x^{-4}y^8\right)^2$

34.) $(9a^3b^5)(-4a^3b^7)^2$

35.) $\left(\frac{4\text{m}^4\text{n}^{-3}\text{p}^2}{6\text{m}^2\text{n}^2}\right)^{-2}$

36.) $\left(4t^3v^2\right)\left(-8tv^5\right)\left(u^0\right)$

37.) $\left(\frac{3x}{y^{-3}}\right)^3\left(\frac{5x^{-10}yz^2}{2x^{-1}y^3}\right)^{-2}$

Lesson 1-2: Rewriting between rational and radical notation and Simplifying Radicals

Standard: MGSE9-12.N.RN.2 - Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Learning Target: R1: ____

Property	Rule	Helpful Hint	Example
Radical to Rational Notation	$\sqrt[n]{a} = a^{\frac{1}{n}}$	Roots are at the bottom of trees	$\sqrt[3]{\mathbf{11}}$ $\sqrt[5]{\boldsymbol{x}^{\mathbf{3}}}$
Rational to Radical Notation	$a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{a}\right)^m$	Highest part of exponent stays high	$6^{1/3}$ $x^{5/2}$

More Examples and Practice – Converting to Rational Exponent Notation:

1. $\sqrt[4]{81}$

2. $\sqrt[5]{32}$

3. $(\sqrt{36})^3$

4. $\left(\sqrt[4]{x}\right)^2$

More Examples and Practice – Converting to Radical Notation:

5. $49^{1/2}$

6. $7^{1/4}$

7. $5^{2/5}$

8. $x^{2/3}$

Property	Rule	Helpful Hint	Example
Product Rule	$\sqrt[a]{x} \cdot \sqrt[a]{y} = \sqrt[a]{xy}$	Same index… multiply terms	$\sqrt{10} \cdot \sqrt{x} =$
Quotient Rule	$\dfrac{\sqrt[a]{x}}{\sqrt[a]{y}} = \sqrt[a]{\dfrac{x}{y}}$	Same index… divide terms	$\dfrac{\sqrt{10}}{\sqrt{2}} =$

More Examples and Practice:

1. $\sqrt{48}$

2. $\sqrt{7x} \cdot \sqrt{2y}$

3. $\dfrac{\sqrt[3]{12x^2}}{\sqrt[3]{4x}}$

4. $\sqrt{\dfrac{144}{25}}$

5. $\sqrt{\dfrac{x^6}{121}}$

6. $\sqrt{16x^8y^{12}}$

7. $\sqrt[4]{64a^8b^{40}}$

8. $\sqrt[3]{27a^{21}b^6}$

Algebra II - Unit 1: Revisiting Quadratics

Lesson 1-3: Complex Operations and Equations

Standard: MGSE9-12.N.CN.1 - Understand there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi where a and b are real numbers.

Learning Target: K2: ____

Introduction of "i"

The imaginary unit "i" is defined as $i = \sqrt{-1}$ and are used to find the square root of negative numbers.

If $i = \sqrt{-1}$, what is i^2?

Standard Form:

The standard form of a complex number is written as a number a + bi, where "a" and "b" are real numbers.

Ex 1: 5i – 4

Ex 2: 2i + 5 + 4i

Ex 3: $2i^2 - 6i$

Taking the Square Root of Negative Numbers:

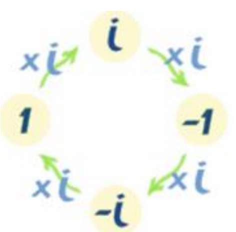
Step 1: Rewrite the radical as $\sqrt{-1} \cdot \sqrt{number}$
Step 2: Rewrite $\sqrt{-1}$ as "i" and then simplify the remaining radical
Step 3: Rewrite in standard form

1. $\sqrt{-25}$

2. $\sqrt{-49}$

3. $\sqrt{-32}$

The Cycle of "i":



- i × i = **-1**,
- then **-1** × i = -i,
- then -i × i = **1**,
- then **1** × i = **i** (back to i again!)

$i^0 = 1$	$i^4 = 1$
$i^1 = i$	$i^5 = i$
$i^2 = -1$	$i^6 = -1$
$i^3 = -i$	$i^7 = -i$

How to Simplify when "i" is raised to higher powers:

STEP 1: Divide the power by 4
STEP 2: Look at what is after the decimal and think money. Ask yourself how many quarters you would have and then refer back to the chart above.

Ex. i^9 $\quad \frac{9}{4} = 2.\boxed{25}$ **There would be 1 quarter so it would be i^1 or just $\boxed{i}$**

1. i^{12}

2. i^{13}

3. i^{20}

4. i^{22}

5. i^{46}

6. i^{99}

7. i^{1000}

8. i^{825}

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Operations with Complex Numbers

Standard: MGSE9-12.N.CN.2 - Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Learning Target: K3: ____

Adding and Subtracting Complex Numbers (write the answer in standard form (a + b*i*):

Ex: $2i + 5i$

Ex: $(3 + 5i) - (2 - 6i)$

Ex: $(2 - 3i) + (8 - 2i)$

You Try:

1. $5i - 8i$

2. $(3 + 5i) + (2 - 6i)$

3. $(2 - 3i) - (8 - 2i)$

Multiplying Complex Numbers:

Step 1: Multiply using your preferred method
Step 2: Anywhere there is an "i^2" replace it with (-1)
Step 3: Simply and write the answer in standard form $(a + bi)$

Ex: $2i \cdot 5i$

Ex: $(2 + 4i)(7 - 8i)$

Ex: $(3 + 4i)^2$

You Try:

4. $5i \cdot -8i$

5. $(3 + 5i)(2 - 6i)$

6. $(2 - 3i)(8 - 2i)$

7. $(3 + 5i)(3 - 5i)$

8. $(2 + 7i)^2$

8. $3(2 + 4i) + 2(3 - 5i)$

Algebra II - Unit 1: Revisiting Quadratics

Standard: MGSE9-12.N.CN.3 - Find the conjugate of a complex number; use the conjugate to find the quotient of complex numbers.

Learning Target: R2: ____

____

Finding a Complex Conjugate (write the answer in standard form (a + b*i*):

Step 1: Change the sign/operation of the complex number. Do <u>not</u> change the sign of the real number.

Ex: $4 + 6i$

Ex: $-5 - 3i$

Ex: $8i$

Ex: 6

Dividing Complex Numbers

Step 1: Find the complex conjugate of the denominator
Step 2: Multiply the numerator and denominator by the complex conjugate of the denominator
Step 3: Simply and write the answer in standard form (a + bi)

Ex: $\frac{10i}{6i}$

Ex: $\frac{2+5i}{3i}$

Ex: $\frac{6-5i}{4-i}$

9. $\frac{3+2i}{5+3i}$

10. $\frac{5+6i}{2+3i}$

11. $\frac{-4+2i}{-3+2i}$

Algebra II - Unit 1: Revisiting Quadratics

Complex Numbers - Additional Practice

Simplify the following:

1. $\sqrt{-7}$

2. $\sqrt{-81}$

3. $\sqrt{-21}$

4. i^{941}

5. i^{15}

6. i^{304}

7. (-3 + 4i) + (6 + 2i)

8. (12 – 8i) – (6 – 6i)

9. (2 – 4i) + (-8 + 9i)

10. (3 + 8i) + (9 – 4i)

11. (12 + 4i) – (8 + 10i)

12. (8 – 4i) – (-5 + 6i)

13. 8i · 3i

14. -2i · 3i

15. 5i · -2i

16. 2i(i - 2)

17. -4i(i + 3)

18. 7i(i + 4)

19. (2 – 3i)(4 + 5i)

20. (3 + 5i)(1 + 4i)

21. (-4i – 2i)(2i – 3i)

22. $\frac{2+5i}{-3i}$

23. $\frac{3+7i}{4-2i}$

24. $\frac{5-2i}{1+4i}$

Lesson 1-4: Solving Quadratics by Factoring

Standard: MGSE9-12.N.CN.7 - Solve quadratic equations with real coefficients that have complex solutions by (but not limited to) square roots, completing the square, and the quadratic formula.

Standard: MGSE9-12.A.REI.4b - Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, factoring, completing the square, and the quadratic formula, as appropriate to the initial form of the equation.

Learning Target: R3: ____

Learning Target: R4: ____

Review of Factoring:

STEP 1: Check for a GCF
STEP 2: Check difference of Squares
STEP 3: Check a·c method where a = 1
STEP 4: Check a·c method where a ≠ 1 (grouping method)

Factor the following if possible:

Ex. $15x^2 – 9$

Ex. $x^2 – 49$

Ex. $4x^2 – 64$

You Try:

1. $15x^2 + 3x$

2. $x^2 - 36$

3. $3b^2 - 300$

Factor the following if possible:

Ex. $x^2 – 10x – 24$

Ex. $x^2 – 13x + 30$

Ex. $3g^2 – 24g – 60g$

You Try:

4. $w^2 + 6w - 40$

5. $m^2 - 7m - 60$

6. $2a^2 - 18a + 40$

Factor the following if possible:

Ex. $4x^2 + 12x + 9$

Ex. $2x^2 + 7x - 15$

Ex. $6m^2 + 10m - 24$

You Try:

7. $6x^2 - 1x - 2$

8. $3a^2 - 10a + 8$

9. $2x^2 - 2x - 24$

Solving with Factoring:

Step 1: Put the equation in standard form (It should equal zero)
Step 2: Factor fully
Step 3: Set each factor equal to zero and solve

Ex. $m^2 - 11m + 18 = 0$

Ex. $2t^2 = 16t + 40$

Ex. $r^2 - 6r = 0$

Ex. $h^2 - 25 = 0$

Ex. $2x^2 + 5x - 12 = 0$

You Try:

1. $x^2 - 2x - 8 = 0$
2. $8p^2 - 18 = 0$
3. $2h - 16 = 0$

4. $3x^2 + 3x - 2 = 10 - 2x$

5. $m^2 - 100 = 0$

6. $8x^2 - 6x - 9 = 0$

Standard: MGSE9-12.N.CN.8 - Extend polynomial identities to include factoring with complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.

Learning Target: R4: ____

Factoring with Complex Factors:

Ex. $n^2 + 36$

Ex. $4a^2 + 25$

Ex. $5q^2 + 45$

1. $12t^2 + 3$

2. $x^2 + 100$

3. $2p^2 + 36$

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Lesson 1-5: The Discriminant and the Quadratic Formula

Standard: MGSE9-12.A.REI.4b - Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, factoring, completing the square, and the quadratic formula, as appropriate to the initial form of the equation.

Learning Target: R3: ____

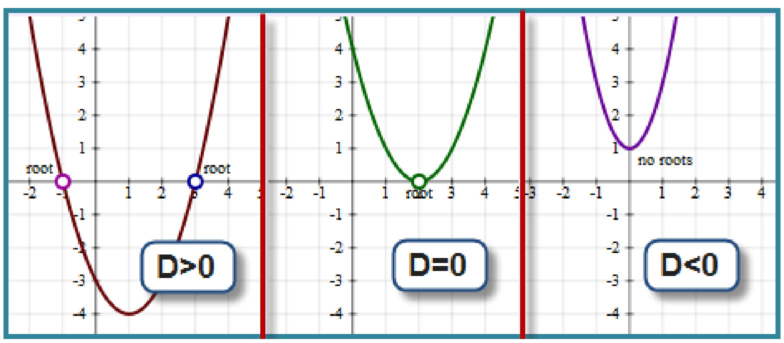
The Discriminant

The discriminant determines the type of solution: Rational or Irrational and Real or Imaginary.

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}} \longrightarrow b^2 - 4ac = \textbf{\textit{discriminant}}$$

If the discriminant is > 0 (positive)	There are 2 Real Solutions and 0 Imaginary Solutions
If the discriminant is = 0	There is 1 Real Solution and 0 Imaginary Solutions
If the discriminant is < 0 (negative)	There are 0 Real Solutions and 2 Imaginary Solutions

Need a picture??



2 Real Solutions **1 Real Solution** **0 Real Solutions**

There are several terms that mean the same thing:
- **Solution**
- **Roots**
- **Zeros**
- **x-intercepts**

Finding the Discriminant:

Step 1: Make sure you are in standard form. $ax^2 + bx + c = 0$
Step 2: Identify a, b, c
Step 3: Substitute into the discriminant formula. $b^2 - 4ac$
Step 4: Use the discriminant to determine the number and type of solutions.

Ex. $2x^2 - 8x - 14 = 0$

Ex. $3x^2 - 15x + 12 = 0$

Ex. $8x^2 - 24x + 18 = 0$

You Try:

1. $3x^2 + 2x + 8 = 0$
2. $3x^2 - 10x - 7 = 0$
3. $x^2 - 6x + 13 = 0$

Algebra II - Unit 1: Revisiting Quadratics

The Quadratic Formula

When using the quadratic formula you are finding solutions which represent the x-intercepts on the graph of a quadratic function.

Standard Form: $y = ax^2 + bx + c$ **Quadratic Formula:** $x = \frac{-b \pm \sqrt{(b)^2 - 4(a)(c)}}{2(a)}$

Ex: $x^2 + 6x + 5 = 0$

Ex: $x^2 - 4x + 4 = 0$

Ex: $2x^2 - 3x + 4 = 0$

You Try:

1. $x^2 + 10x - 19 = 0$
2. $x^2 + 15x - 8 = 0$
3. $2x^2 - 8x + 15 = 0$
4. $3x^2 - 7x - 6 = 0$
5. $2x^2 - 10x - 3 = 0$
6. $3x^2 - 6x + 7 = 0$

Challenge Problems… What is different about these??

7. $3x^2 + 2x = 2x^2 - 1$
8. $2x^2 - 3x = -2$

Algebra II - Unit 1: Revisiting Quadratics

UNIT 1 Reflection **Name:** ______________________ **Period:** ______

Do you feel like you stay on task during class? ______________________

What do you think is your biggest distraction in class? ______________________

____

Do you feel like you are struggling with the content? ______________________

What about this unit did you find to be the easiest? ______________________

____

____

What about this unit did you find to be the most difficult? ______________________

____

____

How can I help you be more successful? ______________________

____

____