NAME: HIRSCH PERIOD: 2nd

Adv. Algebra UNIT 1

Quadratics Revisited

Advanced Algebra - Unit 1: Revisiting Quadratics **Student Name:** ____________________

PRETEST / POST TEST ITEM ANALYSIS **Period:** __________

	PRE TEST		POST TEST	
	Overall Score:		**Overall Score:**	
Learning Targets:	Circle the questions that were correct	Percent Correct: $\frac{\text{\# correct}}{\text{\# of questions}}(100)$	Circle the questions that were correct	Percent Correct: $\frac{\text{\# correct}}{\text{\# of questions}}(100)$
K1: I can extend the properties of integer exponents to rational numbers. (N.RN.1)	4 12		4 12	
R1: I can rewrite expressions involving radical and rational exponents. (N.RN.2)	2 13 14 15		2 13 14 15	
K2: I understand there is a complex number "i" such that $i^2 = -1$. (N.CN.1)	7		7	
K3: I can add, subtract, and multiply complex numbers. (N.CN.2)	3 8 16 19 20		3 8 16 19 20	
K4: I can find the conjugate of a complex number. (N.CN.3)	10		10	
R2: I can use the conjugate to find the quotient of complex numbers. (N.CN.3)	9		9	
R3: I can solve quadratic equations that have complex solutions. (N.CN.7)	11 17		11 17	
R4: I can factor polynomials to include complex solutions. (N.CN.8)	1 18		1 18	
K5: I can solve quadratic equations. (A.REI.4b)	5 6		5 6	

Advanced Algebra - Unit 1: Revisiting Quadratics

Student Name: ____________________ **Period:** ______

Resources can be found at http://hirschalgebra.weebly.com

Graduate Learner Outcome:

As a Henry County graduate, I will be able to reason, describe, and analyze quantitatively using units and number systems to make sense of and solve problems.

As a Henry County graduate, I will be able to create, interpret, use, and analyze patterns of algebraic structures to make sense of problems.

PRE TEST Score:	POST TEST Score:

Standards:	Learning Targets:	Activities:	Performance Task / Quizzes / Test:
Extend the properties of exponents to rational exponents. MGSE9-12.N.RN.1 Explain how the meaning of rational exponents follows from extending the properties of integer exponents to rational numbers, allowing for a notation for radicals in terms of rational exponents. For example, we define 5(1/3) to be the cube root of 5 because we want $[5(1/3)]^3 = 5[(1/3) \times 3]$ to hold, so $[5(1/3)]^3$ must equal 5. MGSE9-12.N.RN.2 - Rewrite expressions involving radicals and rational exponents using the properties of exponents. ______________________	K1: I can extend the properties of integer exponents to rational numbers. (N.RN.1) R1: I can rewrite expressions involving radical and rational exponents. (N.RN.2)	Activities can be found in your unit 1 packet. 1. Lesson 1-1: Simplifying Exponents • Notes • Practice Score: ______ Initial: ______ 2. Lesson 1-2: Rewriting between rational and radical notation and Simplifying Radicals • Notes • Practice Score: ______ Initial: ______ **Quiz Grade:** ______ Remediation (if required): ______________________	**Performance Task:** Based on the daily learning targets from this unit, you will create a series of questions as well as provide the answer and steps taken to solve each problem to show content mastery. This will be completed during our daily warm-up time as a review of prior learning targets and will also be used to create a cumulative math game at the end of this course. **Quizzes:** Quizzes will be given throughout to assess student progress. **Tests:** You will complete a post test at the end of this unit to show mastery.

Perform arithmetic operations with complex numbers.			
MGSE9-12.N.CN.1 - Understand there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi where a and b are real numbers.	K2: I understand there is a complex number "i" such that $i^2 = -1$. (N.CN.1)	3. Lesson 1-3: Complex Operations and Equations • Notes • Practice Score: _______ Initial: _______	
MGSE9-12.N.CN.2 - Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	K3: I can add, subtract, and multiply complex numbers. (N.CN.2)	**Quiz Grade:** _______ Remediation (if required): _______________	
MGSE9-12.N.CN.3 - Find the conjugate of a complex number; use the conjugate to find the quotient of complex numbers.	K4: I can find the conjugate of a complex number. (N.CN.3) R2: I can use the conjugate to find the quotient of complex numbers. (N.CN.3)		
Use complex numbers in polynomial identities and equations.			
MGSE9-12.N.CN.7 - Solve quadratic equations with real coefficients that have complex solutions by (but not limited to) square roots, completing the square, and the quadratic formula.	R3: I can solve quadratic equations that have complex solutions. (N.CN.7)	4. Lesson 1-4: Solving Quadratics by Factoring • Notes • Practice Score: _______ Initial: _______ **Quiz Grade:** _______	
MGSE9-12.N.CN.8 - Extend polynomial identities to include factoring with complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.	R4: I can factor polynomials to include complex solutions. (N.CN.8)		

Solve equations and inequalities in one variable. MGSE9-12.A.REI.4 - Solve quadratic equations in one variable. MGSE9-12.A.REI.4b - Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, factoring, completing the square, and the quadratic formula, as appropriate to the initial form of the equation.	K5: I can solve quadratic equations. (A.REI.4b)	5. Lesson 1-5: The Discriminant and the Quadratic Formula • Notes • Practice Score: ______ Initial: ______ **Quiz Grade:** ______ Remediation (if required): ______________________________	

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Lesson 1-1: Simplifying Exponents

Standard: MGSE9-12.N.RN.1 Explain how the meaning of rational exponents follows from extending the properties of integer exponents to rational numbers, allowing for a notation for radicals in terms of rational exponents.

Learning Target: K1: I can extend the properties of integer exponents to rational numbers.

Properties of Exponents

Property	Rule	Helpful Hint	Example
Product of Powers	$a^m \bullet a^n = a^{m+n}$	$\times \to +$	$x^3 \bullet x^2 =$ x^5
Power of a Power	$(a^m)^n = a^{m \bullet n}$	$\text{Power}^{\text{Power}} \to \times$	$(x^3)^2 =$ x^6
Power of a Product	$(ab)^m = a^m b^m$	Distribute the exponent	$(3x)^4 =$ $3^4x^4 = 81x^4$
Negative Exponent	$a^{-m} = \frac{1}{a^m}$, $a \neq 0$	Turn that frown upside down!	$3^{-2} =$ $\frac{1}{3^2} = \frac{1}{9}$ $\frac{y^2x^{-3}z^5}{a^{-5}b^2c^{-3}} =$ $\frac{y^2z^5a^5c^3}{x^3b^2}$
Zero Exponent	$a^0 = 1$, $a \neq 0$	Zero Power cannot be zero	$4^0 =$ 1
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}$, $a \neq 0$	$\div \to -$	$\frac{x^7}{x^4} =$ x^3
Power of a Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, $b \neq 0$	Distribute the exponent	$\left(\frac{5}{3}\right)^2 =$ $\frac{5^2}{3^2} = \frac{25}{9}$

More Examples and Practice – Simplify the following:

1.) $\dfrac{y^2x^{-3}z^5}{y^{-4}x^6z^7}$

$y^6x^{-9}z^{-2}$

$\dfrac{y^6}{x^9z^2}$

2.) $\dfrac{y^6}{x^9 \cdot z^2}$

It's done

3.) $\dfrac{10a^4b^7}{12a^8b^5}$

$\frac{10}{12} \frac{a^4}{a^8} \frac{b^7}{b^5}$

$\dfrac{5b^2}{6a^4}$

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4.) $(3x^4)(2x^3)$

$6x^7$

5.) $\left(5p^4q^2\right)^2$

$(5)^2(p^4)^2(q^2)^2$

$25p^8q^4$

6.) $\left(4t^3v^2\right)\left(-8tv^5\right)\left(u^0\right)$ (1)

$-32t^4v^7$

7.) $\left(-3m^3n\right)^2\left(2m^2n^4\right)$

$(-3)^2(m^3)^2(n)^2$

$(9m^6n^2)(2m^2n^4)$

$18m^8n^6$

8.) $\dfrac{18\,a^{-3}b^2c^6}{24\,a^2b^5c^4}$

$\dfrac{3c^2}{4a^5b^3}$

9.) $\left(\dfrac{2x^5y^3z^{-2}}{5x^2y^4}\right)^2$

$\left(\dfrac{2x^3}{5yz^2}\right)^2$

$\dfrac{(2)^2(x^3)^2}{(5)^2(y)^2(z^2)^2} = \dfrac{4x^6}{25y^2z^4}$

10.) $\left(6a^2b^5c\right)\left(4a^3b^{-2}c^8\right)$

$24a^5b^3c^9$

11.) $\left(4p^3q^{-2}t^4\right)^2$

$(4)^2(p^3)^2(q^{-2})^2(t^4)^2$

$16p^6q^{-4}t^8$

$\dfrac{16p^6t^8}{q^4}$

12.) $\left(\dfrac{4x^2y^{-3}}{y^{-2}}\right)^{-1}$

$\dfrac{y^{-2}}{4x^2y^{-3}}$

$= \dfrac{y}{4x^2}$

13.) $\left(8x^7y^3\right)\left(3x^{-4}y^8\right)^2$

$(8x^7y^3)(3)^2(x^{-4})^2(y^8)^2$

$(8x^7y^3)(9x^{-8}y^{16})$

$72x^{-1}y^{19}$

$\dfrac{72y^{19}}{x^1}$

14.) $(9a^3b^5)(-4a^3b^7)^2$

$(9a^3b^5)(-4)^2(a^3)^2(b^7)^2$

$(9a^3b^5)(16a^6b^{14})$

$144a^9b^{19}$

15.) $\left(\dfrac{4m^4n^{-3}p^2}{6m^2n^2}\right)^{-2}$

$\left(\dfrac{2m^2p^2}{3n^5}\right)^{-2}$

$\left(\dfrac{(2)^{-2}(m^2)^{-2}(p^2)^{-2}}{(3)^{-2}(n^5)^{-2}}\right)$

$\dfrac{9n^{10}}{4m^4p^4}$

16.) $\left(\dfrac{3x}{y^{-3}}\right)^3\left(\dfrac{5x^{-10}yz^2}{2x^{-1}y^3}\right)^{-2}$

$\dfrac{(3)^3(x)^3}{(y^{-3})^3} \quad \dfrac{(5)^{-2}(x^{-10})^{-2}(y)^{-2}(z^2)^{-2}}{(2)^{-2}(x^{-1})^{-2}(y^3)^{-2}}$

$27x^3y^9 \quad \dfrac{4x^{20}y^6}{25y^2z^4x^2}$

$\dfrac{27x^3y^9 \quad 4x^{18}y^4}{25z^4} = \dfrac{108x^{21}y^{13}}{25z^4}$

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Properties of Exponents Extended to Include Rational Exponents

Simplify the following:

17.) $x^{\frac{1}{5}} \cdot x^{\frac{3}{5}}$ $x^{4/5}$

18.) $\dfrac{a^{\frac{5}{3}}}{a^{\frac{1}{3}}}$ $a^{4/3}$

19.) $\dfrac{m^{\frac{2}{3}} \cdot m^{-\frac{1}{2}}}{m^{\frac{1}{6}}}$ $\dfrac{m^{1/6}}{m^{1/6}} = m^0 = 1$

20.) $\left(b^{\frac{1}{4}}\right)^4$ b^1 b

21.) $\left(p^{\frac{3}{4}}\right)^{\frac{2}{5}}$ $p^{3/10}$

22.) $\dfrac{w^{\frac{1}{6}}}{w^{-\frac{2}{3}}}$ $w^{5/6}$

23.) $7^4 \cdot 7^{12}$ 7^{16}

24.) $t^{-4} \cdot t^{15}$ t^{11}

25.) $3^{2/3} \cdot 3^{3/4}$ $3^{17/12}$

26.) $(-8)^8 \cdot (-8)^{-13}$ $(-8)^{-5} = \dfrac{1}{(-8)^5} = \dfrac{1}{-32768}$

27.) $\dfrac{b^8}{b^6}$ b^2

28.) $\dfrac{b^{1/3}}{b^{1/4}}$ $b^{1/12}$

29.) $\dfrac{11^{14}}{11^{18}}$ $\dfrac{1}{11^4}$ $\dfrac{1}{14641}$

30.) $(9^5)^4$ 9^{20}

31.) $5^{1/5} \cdot 5^{2/3}$ $5^{13/15}$

32.) $(c^2)^9$ c^{18}

33.) $\left(\dfrac{5}{3}\right)^4$ $\dfrac{5^4}{3^4}$ $\dfrac{625}{81}$

34.) $(6^{3/4})^{4/5}$ $6^{3/5}$

35.) $\left(\dfrac{7}{2}\right)^{-3}$ $\left(\dfrac{2}{7}\right)^3$ $\dfrac{2^3}{7^3}$ $\dfrac{8}{343}$

36.) $\dfrac{h^4 x^{-7}}{p^{-5} b^{18}}$ $\dfrac{h^4 p^5}{x^7 b^{18}}$

37.) $\left(\dfrac{4}{5}\right)^{1/2}$ $\dfrac{4^{1/2}}{5^{1/2}}$ $\dfrac{2}{5^{1/2}}$

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Lesson 1-3: Complex Operations and Equations

Standard: MGSE9-12.N.CN.1 - Understand there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi where a and b are real numbers.

Learning Target: K2: I understand there is a complex # "i" such that $i^2 = -1$.

Introduction of "i"

The imaginary unit "i" is defined as $i = \sqrt{-1}$ and are used to find the square root of negative numbers.

If $i = \sqrt{-1}$, what is i^2? $(\sqrt{-1})^2 = -1$

Standard Form:

The standard form of a complex number is written as a number a + bi, where "a" and "b" are real numbers.

Ex 1: $5i - 4$ $\quad -4+5i$

Ex 2: $2i + 5 + 4i$ $\quad 5+6i$

Ex 3: $2i^2 - 6i$ $\quad 2(-1)-6i$ $\quad -2-6i$

Taking the Square Root of Negative Numbers:

Step 1: Rewrite the radical as $\sqrt{-1} \cdot \sqrt{number}$
Step 2: Rewrite $\sqrt{-1}$ as "i" and then simplify the remaining radical
Step 3: Rewrite in standard form

1. $\sqrt{-25}$ $\quad \sqrt{-1}\cdot\sqrt{25}$ $\quad i\cdot 5$ $\quad 5i$

2. $\sqrt{-49}$ $\quad 7i$

3. $\sqrt{-32}$ $\quad 4i\sqrt{2}$

The Cycle of "i":

- i × i = **-1**,
- then **-1** × i = -i,
- then -i × i = **1**,
- then **1** × i = **i** (back to i again!)

$i^0 = 1$	$i^4 = 1$
$i^1 = i$	$i^5 = i$
$i^2 = -1$	$i^6 = -1$
$i^3 = -i$	$i^7 = -i$

How to Simplify when "i" is raised to higher powers:

STEP 1: Divide the power by 4
STEP 2: Look at what is after the decimal and think money. Ask yourself how many quarters you would have and then refer back to the chart above.

Ex. i^9 $\quad \frac{9}{4} = 2.\boxed{25}$ $\quad$ There would be 1 quarter so it would be i^1 or just $\boxed{i}$ $\quad i^9 = i$

1. i^{12} $\quad \frac{12}{4} = 3.\boxed{00}$ $\quad i^0 = 1$

2. i^{13} $\quad \frac{13}{4}$ $\quad 3.\boxed{25}$ $\quad i^1$ $\quad i$

3. i^{20} $\quad 5.\boxed{00}$ $\quad i^0 = 1$

4. i^{22} $\quad \frac{22}{4} = 5.\boxed{50}$ $\quad i^2 = -1$

5. i^{46} $\quad 11.\boxed{50}$ $\quad i^2$ $\quad -1$

6. i^{99} $\quad 24.\boxed{75}$ $\quad i^3 = -i$

7. i^{1000} $\quad 250$ $\quad i^0 = 1$

8. i^{825} $\quad \frac{825}{4}$ $\quad 206.\boxed{25}$ $\quad i^1 = i$

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Operations with Complex Numbers

Standard: MGSE9-12.N.CN.2 - Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Learning Target: K3: I can add, subtract, and multiply complex #'s.

Adding and Subtracting Complex Numbers (write the answer in standard form (a + b*i*):

Ex: $2i + 5i$ — $7i$

Ex: $(3 + 5i) - (2 - 6i)$
$3+5i-2+6i$
$1+11i$

Ex: $(2 - 3i) + (8 - 2i)$
$10-5i$

You Try:

1. $5i - 8i$
$-3i$

2. $(3 + 5i) + (2 - 6i)$
$5-i$

3. $(2 - 3i) - (8 - 2i)$
$2-3i-8+2i$
$-6-i$

Multiplying Complex Numbers:

Step 1: Multiply using your preferred method
Step 2: Anywhere there is an "i^2" replace it with (–1)
Step 3: Simply and write the answer in standard form (a + bi)

Ex: $2i \cdot 5i$
$10i^2$
$10(-1)$
-10

Ex: $(2 + 4i)(7 - 8i)$
$14-16i+28i-32i^2$
$14+12i-32(-1)$
$14+12i+32$
$46+12i$

Ex: $(3 + 4i)^2$ — $(3+4i)(3+4i)$
$9+12i+12i+16i^2$
$9+24i+16(-1)$
$9+24i-16$
$-7+24i$

You Try:

4. $5i \cdot -8i$
$-40i^2$
$-40(-1)$
40

5. $(3 + 5i)(2 - 6i)$
$6-18i+10i-30i^2$
$6-8i+30$
$36-8i$

6. $(2 - 3i)(8 - 2i)$
$16-4i-24i+6i^2$
$16-28i-6$
$10-28i$

7. $(3 + 5i)(3 - 5i)$
$9-\cancel{15i}+\cancel{15i}-25i^2$
$9+25$
34

8. $(2 + 7i)^2$
$(2+7i)(2+7i)$
$4+14i+14i+49i^2$
$4+28i-49$
$-45+28i$

8. $3(2 + 4i) + 2(3 - 5i)$
$6+12i+6-10i$
$12+2i$

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Standard: MGSE9-12.N.CN.3 - Find the conjugate of a complex number; use the conjugate to find the quotient of complex numbers.

Learning Target: K4: I can find the conjugate of a complex #.

Learning Target: R2: I can use the conjugate to find the quotient of complex #'s.

Finding a Complex Conjugate (write the answer in standard form (a + b*i*):

Step 1: Change the sign/operation of the complex number. Do not change the sign of the real number.

Ex: $4+6i$ $\quad 4-6i$

Ex: $-5-3i$ $\quad -5+3i$

Ex: $8i$ $\quad -8i$

Ex: 6 $\quad 6$

Dividing Complex Numbers

Step 1: Find the complex conjugate of the denominator
Step 2: Multiply the numerator and denominator by the complex conjugate of the denominator
Step 3: Simply and write the answer in standard form (a + bi)

Ex: $\frac{10i}{6i}\cdot\frac{-6i}{-6i}$

$-6i$

$\frac{-60i^2(-1)}{-36i^2(-1)}$

$\frac{60}{36}=\frac{5}{3}$

Ex: $\frac{2+5i}{3i}\cdot\frac{-3i}{-3i}$

$-3i$

Top $-3i(2+5i)$

$-6i-15i^2(-1)$

$-6i+15$

$15-6i$

Bottom $3i(-3i)$

$-9i^2(-1) = 9$

$\frac{15-6i}{9}$

Ex: $\frac{6-5i}{4-i}$

$4+i$

$\boxed{N}$ $(6-5i)(4+i)$

$24+6i-20i-5i^2(-1)$

$24-14i+5 = 29-14i$

$\boxed{D}$ $(4-i)(4+i)$

$16+\cancel{4i}-\cancel{4i}-1i^2(-1)$

$16+1=17$

$\boxed{\frac{29-14i}{17}}$

9. $\frac{3+2i}{5+3i}$

$5-3i$

$\boxed{N}$

	3	$2i$
5	15	$10i$
$-3i$	$-9i$	$-\cancel{6i^2}$ 6

$21+1i$

$\boxed{D}$

	5	$3i$
5	25	$\cancel{15i}$
$-3i$	$\cancel{-15i}$	$-\cancel{9i^2}$ 9

34

$\frac{21+i}{34}$

10. $\frac{5+6i}{2+3i}$

$2-3i$

$\boxed{N}$ $(5+6i)(2-3i)$

$10-15i+12i-\cancel{18i^2}$ $+18$

$28-3i$

$\boxed{D}$ $(2+3i)(2-3i)$

$4-\cancel{6i}+\cancel{6i}-\cancel{9i^2}$ $+9$

13

$\boxed{\frac{28-3i}{13}}$

11. $\frac{-4+2i}{-3+2i}$

$-3-2i$

$\boxed{N}$

	-4	$2i$
-3	12	$-6i$
$-2i$	$8i$	$-\cancel{4i^2}$

4

$16+2i$

$\boxed{D}$

	-3	$2i$
-3	9	$\cancel{-6i}$
$-2i$	$\cancel{6i}$	$\cancel{-4i^2}$

4

13

$\boxed{\frac{16+2i}{13}}$

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Complex Numbers - Additional Practice

Simplify the following:

1. $\sqrt{-7}$ $i\sqrt{7}$

2. $\sqrt{-81}$ $9i$

3. $\sqrt{-21}$ $i\sqrt{21}$

4. $i^{941} = i^1 = i$

5. $i^{15} = i^3 = -i$

6. $i^{304} = i^0 = 1$

7. (-3 + 4i) + (6 + 2i)
$3+6i$

8. (12 – 8i) – (6 – 6i)
$12-8i-6+6i$
$6-2i$

9. (2 – 4i) + (-8 + 9i)
$-6+5i$

10. (3 + 8i) + (9 – 4i)
$12+4i$

11. (12 + 4i) – (8 + 10i)
$12+4i-8-10i$
$4-6i$

12. (8 – 4i) – (-5 + 6i)
$8-4i+5-6i$
$13-10i$

13. 8i · 3i
$24i^2 = 24(-1)$
-24

14. -2i · 3i
$-6i^2 = -6(-1)$
6

15. 5i · -2i
$-10i^2 = -10(-1)$
10

16. 2i(i - 2)
$2i^2-4i$
$2(-1)-4i = -2-4i$

17. -4i(i + 3)
$-4i^2-12i$
$-4(-1)-12i$
$4-12i$

18. 7i(i + 4)
$7i^2+28i$
$7(-1)+28i$
$-7+28i$

19. (2 – 3i)(4 + 5i)
$8+10i-12i-15i^2$
$8-2i+15$
$23-2i$

20. (3 + 5i)(1 + 4i)
$3+12i+5i+20i^2$
$3+17i-20$
$-17+17i$

21. (-4i – 2i)(2i – 3i)
$(-6i)(-i)$
$6i^2 = 6(-1)$
$= -6$

22. $\frac{2+5i}{-3i} \cdot \frac{3i}{3i}$
$\frac{3i(2+5i)}{3i(-3i)}$
$\frac{6i+18i^2}{-9i^2}$ -15 9
$\frac{-15+6i}{9} = \frac{-5+2i}{3}$

23. $\frac{3+7i}{4-2i}$ $4+2i$
$(3+7i)(4+2i)$
$12+6i+28i+14i^2$
$12+34i-14$
$-2+34i$
$(4-2i)(4+2i)$
$16+\cancel{8i}-\cancel{8i}-4i^2$
$16+4$
20
$\frac{-2+34i}{20} = \frac{-1+17i}{10}$

24. $\frac{5-2i}{1+4i}$ $1-4i$
$(5-2i)(1-4i)$
$5-20i-2i+8i^2$
$5-22i-8$
$-3-22i$
$(1+4i)(1-4i)$
$1-4i+4i-16i^2$
$1+16$
17
$\frac{-3-22i}{17}$

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Lesson 1-4: Solving Quadratics by Factoring

Standard: MGSE9-12.N.CN.7 - Solve quadratic equations with real coefficients that have complex solutions by (but not limited to) square roots, completing the square, and the quadratic formula.

Standard: MGSE9-12.A.REI.4b - Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, factoring, completing the square, and the quadratic formula, as appropriate to the initial form of the equation.

Learning Target: R3: I can solve quadratic equations with complex solutions.

Learning Target: K5: I can solve quadratic equations.

Review of Factoring:

STEP 1: Check for a GCF
STEP 2: Check difference of Squares – subtraction, perfect squares
STEP 3: Check a·c method where a = 1
STEP 4: Check a·c method where a ≠ 1 (grouping method)

Factor the following if possible:

Ex. $15x^2 - 9$

$\frac{15x^2}{3} - \frac{9}{3}$

$3(5x^2 - 3)$

Ex. $x^2 - 49$

x x 7 -7

$(x+7)(x-7)$

Ex. $4x^2 - 64$

$\frac{4x^2}{4} - \frac{64}{4}$

$4(x^2 - 16)$

x x 4 -4

$4(x+4)(x-4)$

You Try:

1. $12w^2 + 27$

$\frac{12w^2}{3} + \frac{27}{3}$

$3(4w^2 + 9)$

2. $x^2 - 36$

x x 6 -6

$(x+6)(x-6)$

3. $3b^2 - 300$

$\frac{3b^2}{3} - \frac{300}{3}$

$3(b^2 - 100)$

b b 10 -10

$3(b+10)(b-10)$

4. $m^2 + 9$

cannot factor

"prime"

5. $15x^2 + 3x$

$\frac{15x^2}{3x} + \frac{3x}{3x}$

$3x(5x + 1)$

Factor the following if possible:

Ex. $x^2 - 10x - 24$

$a \cdot c = -24$

4 6
3 8
2 -12
1 24

$(x+2)(x-12)$

Ex. $x^2 - 13x + 30$

$a \cdot c = 30$

1 30
5 6
-3 -10

$(x-3)(x-10)$

Ex. $3g^2 - 24g - 60g$

$\frac{3g^2}{3g} - \frac{24g}{3g} - \frac{60g}{3g}$

$3g(g - 8 - 20)$

$3g(g - 28)$

You Try:

6. $w^2 + 6w - 40$

$a \cdot c = -40$

1 40
2 20
4 −10
−4 10

$(x-4)(x+10)$

$(w-4)(w+10)$

7. $m^2 - 7m - 60$

$a \cdot c = -60$

3 20
4 15
5 −12

$(m+5)(m-12)$

8. $2a^2 - 18a + 40$

$\frac{2a^2}{2} - \frac{18a}{2} + \frac{40}{2}$

$2(a^2 - 9a + 20)$

$a \cdot c = 20$

4 5
−4 −5

$2(a-4)(a-5)$

Factor the following if possible:

Ex. $4x^2 + 12x + 9$

$a \cdot c$
$4 \cdot 9$
36

6 6

$\frac{4x^2}{2x} + \frac{6x}{2x} \Big| + \frac{6x}{3} + \frac{9}{3}$

$2x(2x+3)\ 3(2x+3)$

$(2x+3)(2x+3)$

or

$(2x+3)^2$

Ex. $2x^2 + 7x - 15$

$a \cdot c$
$2 \cdot -15$
-30

3 −10
−3 10

$\frac{2x^2}{x} - \frac{3x}{x} \Big| + \frac{10x}{5} - \frac{15}{5}$

$x(2x-3)\ 5(2x-3)$

$(2x-3)(x+5)$

Ex. $6m^2 + 10m - 24$

$\frac{6m^2}{2} + \frac{10m}{2} - \frac{24}{2}$

$2(3m^2 + 5m - 12)$

$a \cdot c$
-36

4 −9
−4 9

$\frac{3m^2}{m} - \frac{4m}{m} \Big| + \frac{9m}{3} - \frac{12}{3}$

$m(3m-4)\ 3(3m-4)$

$2(3m-4)(m+3)$

You Try:

9. $6x^2 - 1x - 2$

$a \cdot c$
$6 \cdot -2$
-12

3 −4

$\frac{6x^2}{3x} + \frac{3x}{3x} \Big| - \frac{4x}{-2} - \frac{2}{-2}$

$3x(2x+1) - 2(2x+1)$

$(2x+1)(3x-2)$

10. $3a^2 - 10a + 8$

24

2 −12
−4 −6

$\frac{3a^2}{a} - \frac{4a}{a} \Big| - \frac{6a}{-2} + \frac{8}{-2}$

$a(3a-4) - 2(3a-4)$

$(3a-4)(a-2)$

11. $2x^2 - 2x - 24$

$\frac{2x^2}{2} - \frac{2x}{2} - \frac{24}{2}$

$2(x^2 - x - 12)$

$a \cdot c$ 1 −12
−12

−4 3

$2(x-4)(x+3)$

Advanced Algebra - Unit 1: Revisiting Quadratics

Lesson 1-5: The Discriminant and the Quadratic Formula

Standard: MGSE9-12.A.REI.4b - Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, factoring, completing the square, and the quadratic formula, as appropriate to the initial form of the equation.

Learning Target: K5: I can solve quadratic equations.

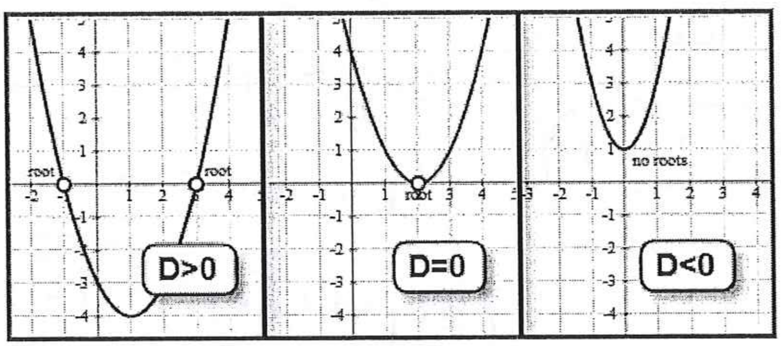
The Discriminant

The discriminant determines the type of solution: Rational or Irrational and Real or Imaginary.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \longrightarrow b^2 - 4ac = \textit{discriminant}$$

If the discriminant is > 0 (positive) — There are 2 Real Solutions and 0 Imaginary Solutions
If the discriminant is = 0 — There is 1 Real Solution and 0 Imaginary Solutions
If the discriminant is < 0 (negative) — There are 0 Real Solutions and 2 Imaginary Solutions

Need a picture??



2 Real Solutions **1 Real Solution** **0 Real Solutions**

There are several terms that mean the same thing:
* **Solution**
* **Roots**
* **Zeros**
* **x-intercepts**

Finding the Discriminant:

Step 1: Make sure you are in standard form. $ax^2 + bx + c = 0$
Step 2: Identify a, b, c
Step 3: Substitute into the discriminant formula. $b^2 - 4ac$
Step 4: Use the discriminant to determine the number and type of solutions.

Ex. $2x^2 - 8x - 14 = 0$
$a=2$ $b=-8$ $c=-14$
$(-8)^2 - 4(2)(-14)$
$d = 176$ 2 real sol

Ex. $3x^2 - 15x + 12 = 0$
$a=3$ $b=-15$ $c=12$
$(-15)^2 - 4(3)(12)$
$d = 81$ 2 Real sol

Ex. $8x^2 - 24x + 18 = 0$
$a=8$ $b=-24$ $c=18$
$(-24)^2 - 4(8)(18)$
$d = 0$ 1 real sol

You Try:

1. $3x^2 + 2x + 8 = 0$
$a=3$ $b=2$ $c=8$
$(2)^2 - 4(3)(8)$
$d = -92$ 0 real sol 2 imaginary

2. $3x^2 - 10x - 7 = 0$
$a=3$ $b=-10$ $c=-7$
$(-10)^2 - 4(3)(-7)$
$d = 184$ 2 Real sol

3. $x^2 - 6x + 13 = 0$
$a=1$ $b=-6$ $c=13$
$(-6)^2 - 4(1)(13)$
$d = -16$ 0 real sol 2 imaginary

Advanced Algebra - Unit 1: Revisiting Quadratics

The Quadratic Formula

When using the quadratic formula you are finding solutions which represent the x-intercepts on the graph of a quadratic function.

Standard Form: y = ax² + bx + c   **Quadratic Formula:** $x = \frac{-b \pm \sqrt{(b)^2 - 4(a)(c)}}{2(a)}$

opp b

Ex: $x^2 + 6x + 5 = 0$

$a=1\ b=6\ c=5$

$x = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(5)}}{2(1)}$

$x = \frac{-6 \pm \sqrt{16}}{2}$ + -1, − -5

Ex: $x^2 - 4x + 4 = 0$

$a=1\ b=-4\ c=4$

$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(4)}}{2(1)}$

$x = \frac{4 \cancel{\pm \sqrt{0}}}{2} = \frac{4}{2} = 2$

Ex: $2x^2 - 3x + 4 = 0$

$a=2\ b=-3\ c=4$

$x = \frac{3 \pm \sqrt{(-3)^2 - 4(2)(4)}}{2(2)}$

$x = \frac{3 \pm \sqrt{-23}}{4}$ + $\frac{3 + i\sqrt{23}}{4}$, − $\frac{3 - i\sqrt{23}}{4}$

You Try:

1. $x^2 + 10x - 19 = 0$

$a=1\ b=10\ c=-19$

$x = \frac{-10 \pm \sqrt{(10)^2 - 4(1)(-19)}}{2(1)}$

$x = \frac{-10 \pm \sqrt{176}}{2}$ + 1.633, − -11.633

2. $x^2 + 15x - 8 = 0$

$a=1\ b=15\ c=-8$

$x = \frac{-15 \pm \sqrt{(15)^2 - 4(1)(-8)}}{2(1)}$

$x = \frac{-15 \pm \sqrt{257}}{2}$ + 0.516, − -15.516

3. $2x^2 - 8x + 15 = 0$

$a=2\ b=-8\ c=15$

$x = \frac{8 \pm \sqrt{(-8)^2 - 4(2)(15)}}{2(2)}$

$x = \frac{8 \pm \sqrt{-56}}{4}$

$x = \frac{8 \pm 2i\sqrt{14}}{4} = \frac{4 \pm i\sqrt{14}}{2}$

4. $3x^2 - 7x - 6 = 0$

$a=3\ b=-7\ c=-6$

$x = \frac{7 \pm \sqrt{(-7)^2 - 4(3)(-6)}}{2(3)}$

$x = \frac{7 \pm \sqrt{121}}{6}$ + 3, − -0.667

5. $2x^2 - 10x - 3 = 0$

$a=2\ b=-10\ c=-3$

$x = \frac{10 \pm \sqrt{(-10)^2 - 4(2)(-3)}}{2(2)}$

$x = \frac{10 \pm \sqrt{124}}{4}$ + 5.284, − -0.284

6. $3x^2 - 6x + 7 = 0$

$a=3\ b=-6\ c=7$

$x = \frac{6 \pm \sqrt{(-6)^2 - 4(3)(7)}}{2(3)}$

$x = \frac{6 \pm \sqrt{-48}}{6} = \frac{6 \pm 4i\sqrt{3}}{6}$

$= \frac{3 \pm 2i\sqrt{3}}{3}$

Challenge Problems… What is different about these??

7. $3x^2 + 2x = 2x^2 - 1$

$-2x^2 + 1 \quad -2x^2 + 1$

$x^2 + 2x + 1 = 0$

$(x+1)(x+1) = 0$

$x + 1 = 0$

$-1 \quad -1$

$x = -1$

8. $2x^2 - 3x = -2$

$+2 \quad +2$

$2x^2 - 3x + 2 = 0$

$a=2\ b=-3\ c=2$

$x = \frac{3 \pm \sqrt{(-3)^2 - 4(2)(2)}}{2(2)}$

$x = \frac{3 \pm \sqrt{-7}}{4} = \frac{3 \pm i\sqrt{7}}{4}$