Algebra II UNIT 2

Operations With Polynomials

WHAT ARE YOU LEARNING?

Henry County Graduate Learner Outcomes:

- As a Henry County graduate, I will be able to create, interpret, use, and analyze patterns of algebraic structures to make sense of problems.
- As a Henry County graduate, I will be able to use functions to interpret and analyze a variety of contexts.

Georgia Standards of Excellence:

Lesson 2-1 – Basic Polynomial Operations

MGSE9-12.A.APR.1 Add, subtract, and multiply polynomials; understand that polynomials form a system analogous to the integers in that they are closed under these operations.

Lesson 2-2 – Binomial Theorem / Pascal's Triangle

MGSE9-12.A.APR.5 Know and apply that the Binomial Theorem gives the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.

Lesson 2-3 – Synthetic and Long Division

MGSE9-12.A.APR.6 Rewrite simple rational expressions in different forms using inspection, long division, or a computer algebra system; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x).

Lesson 2-4 – Composition of Functions

MGSE9-12.F.BF.1 Write a function that describes a relationship between two quantities.

MGSE9-12.F.BF.1b Combine standard function types using arithmetic operations in contextual situations (Adding, subtracting, and multiplying functions of different types).

MGSE9-12.F.BF.1c Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

Lesson 2-5 – Inverses of Functions

MGSE9-12.F.BF.4 Find inverse functions.

MGSE9-12.F.BF.4a Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2(x^3)$ or f(x) = (x+1)/(x-1) for $x \neq 1$.

MGSE9-12.F.BF.4b Verify by composition that one function is the inverse of another.

MGSE9-12.F.BF.4c Read values of an inverse function from a graph or a table, given that the function has an inverse.

WHY ARE YOU LEARNING THIS?

Level 3 Performance Task:

Polynomial Puzzle

Towards the end of this unit you will be given a puzzle with various polynomial operations. You will use the skills you learn in this unit to solve the puzzle.

WHAT IS YOUR GOAL FOR THIS UNIT?

Unit Goal:

I scored a _____ on my pretest.

My goal is to score a _____ or higher on the end of unit test.

To achieve this goal I will ______

HOW WILL YOU KNOW WHEN YOU'VE MASTERED THIS? SHOW ME THE EVIDENCE!

Data Analysis:

Pre-Test Score _____ Post-Test Score _____

Learning Targets:	Pre-Test Score	Quiz Score	Post-Test Score
Prerequisite Skills			
K1: I can add, subtract, and multiply polynomials. (A.APR.1)			
R1: I can apply the binomial theorem to expand a binomial. (A.APR.5)			
R2: I can rewrite simple rational expressions in different forms including the use of synthetic and long division. (A.APR.6)			
K2: I can combine standard functions by addition, subtraction, and multiplication. (F.BF.1b)			
R4: I can compose functions. (F.BF.1c)			
R5: I can find inverse functions. (F.BF.4 and F.BF.4a)			
R6: I can use composite functions to verify inverse function relationships. (F.BF.4b)			
K3: I can read values of an inverse function from a graph or table. (F.BF.4c)			

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LEARNING ACTIVITIES		
Lesson 2-1 – Basic Polynomial Operations		
K1: I can add, subtract, and multiply polynomials. (A.A	APR.1)	
Complete guided notes with teacher Complete 2-1 practice	OR	Watch video lesson and take notes
Lesson 2-2 – Binomial Theorem / Pascal's Triangle		
R1: I can apply the binomial theorem to expand a bin	omial. (A	.APR.5)
Complete guided notes with teacher Complete 2-2 practice	OR	Watch video lesson and take notes
Lesson 2-3 – Synthetic and Long Division		
R2: I can rewrite simple rational expressions in differe (A.APR.6)	ent forms	s including the use of synthetic and long division.
Complete guided notes with teacher Complete 2-3 practice	OR	Watch video lesson and take notes
Quiz 2-1 to 2-3		
Lesson 2-4 – Composition of Functions		
R3: I can write a function that describes a relationship K2: I can combine standard functions by addition, sub R4: I can compose functions. (F.BF.1c)		
Complete guided notes with teacher Complete 2-4 practice	OR	Watch video lesson and take notes
Lesson 2-5 – Inverse of Functions		
R5: I can find inverse functions. (F.BF.4 and F.BF.4a) R6: I can use composite functions to verify inverse fun K3: I can read values of an inverse function from a gra		
Complete guided notes with teacher Complete 2-5 practice	OR	Watch video lesson and take notes
Quiz 2-4 to 2-5		
UNIT 2 Assessments		
Complete Unit 2 Performance Task (com Complete Unit 2 Review Guide Complete Unit 2 Test Complete Unit 2 Reflection	pleted aft	er lesson 3)

Lesson 2-1: Basic Polynomial Operations

Learning Target: K1: _____

Name	Examples	Non-Examples
Monomial (one term)	1. $3x^4$ 2. a^2 3. 5	degree:4 or quartic1. $2x^{-4}$ degree:2 or quadratic2. $5\sqrt{m}$ degree:0 or constant3. $3t^{\frac{2}{3}}$
Binomial (two terms)	1. $2n^3 - n$ 2. $p - 3$ 3. $-3a^3b^4 + a^4b^5$	degree:3 or cubic degree:1 or linear(monic) degree:9 or nonic 2. $\sqrt{C^3 - 2}$
Trinomial (three terms)		degree:3 or cubic 1. $x^{-3} + 2x - 5$ degree:5 or quintic 2. $2^{x} + 3x - 5$
Polynomial (one or more terms)	2. $5y^6$	degree:4 or quartic degree:6 or sextic1. $3q^3 + \frac{p}{q}$ 2. $2^x + 3\sqrt{x}$

Classifying Polynomials

Classifying Polynomials by the number of terms:

1 Term:	Monomial	3 Terms:	Trinomial
2 Terms:	Binomial	1+ Terms:	Polynomial

Classifying Polynomials by the degree: The degree of the polynomial is the same as the term with the highest degree.

Degree of 0:	Constant	Degree of 3:	Cubic
Degree of 1:	Linear	Degree of 4:	Quartic
Degree of 2:	Quadratic	Degree of 5:	Quintic

You Try:

Polynomial	Number of Terms	Classification	Degree	Classification
f(x) = 3x - 1	2	Binomial	1	Linear
$f(x) = -x^5$				
$f(x) = 8x^3 + 125$				
$f(x) = x^4 + 10x^2 + 16$				
$f(x) = x^2 - 2x + 1$				
f(x) = 2				

Review of Adding, Subtracting, and Multiplying Polynomials

Expand and Simplify (put your answer in standard form):

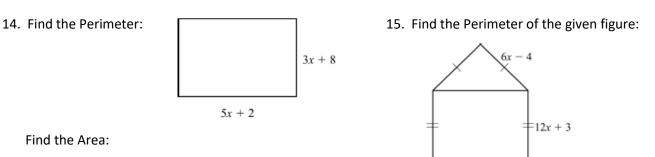
1.
$$(7x + 3) - (2 - 2x)$$
 2. $(5x^3 - 3x^4 - 2x - 9x^2 - 2) + (3x^3 + 2x^2 - 5x - 7)$
 3. $3(x + 4) + 8x$

 4. $-2(3x + 2y) - (5x - 6y) + 2x - 7$
 5. $(2x^2 + 5x) - (6x^2 - 2x)$
 6. $(2x^3 + 5x - 8) + (5x^3 - 9x^2 - 11x + 5)$

 7. $(x + 3)(x + 5)$
 8. $(2x - 5)^2$
 9. $4y^2(y^2 + 2y)$

10. $-6y^{2}(3y^{2} - 2y - 7)$ 11. (2x + 3)(3x - 5) 12. $(2a + 3)(a^{2} + 2a - 4)$

13. $(x-2)(x^2+4)(2x+3)$



14x + 13

More Practice - Expand and Simplify (put your answer in standard form):

16.
$$(3x^2 + 2x - 5) + (x^2 + 3x + 5)$$

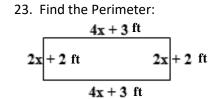
17. $(4x^5 - 3x^2 + 8) - (2x^5 + 2x^2 - 1)$

18.
$$(x + 1) (2x + 3)$$
 19. $(2x^4 + 3x^2 + 7) + (2x^3 - 4)$

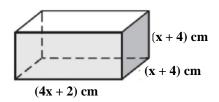
20. $(2x-5)(3x^2-4x+6)$ 21. $(x-4)^3$

22. Find the Area:

(4x + 5) in (4x + 5) in



24. Find the volume:



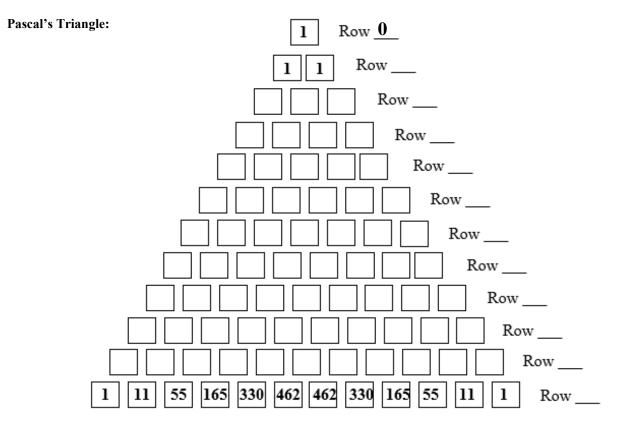
Lesson 2-2: Binomial Theorem – Pascal's Triangle

Learning Target: R1: _____

What if you were asked to multiply $(x - 6)^5$ or even worse $(x - 6)^{13}$?

You'd be there a while right?? Wouldn't you be interested in learning a quicker way??

Pascal's Triangle and the Binomial Theorem can help!!!



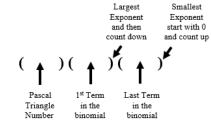
Binomial Theorem:

1. Think back to our previous lesson.... $(x + 2)^3$

We multiplied using distribution.

- (x + 2)(x + 2)(x + 2)(x + 2)(x² + 2x + 2x + 4) (x + 2)(x² + 4x + 4) X³ + 4x² + 4x + 2x² + 8x + 8 X³ + 6x² + 12x + 8
- 2. Let's try it using Pascal's Triangle and the Binomial Theorem?

(x + 2)³



 What about (x + 3)⁵? Would you want to multiply this all the way out using distribution? You can but let's try it using Pascal's Triangle and the Binomial Theorem.

Notice the number of groupings... it is always 1 more than the degree.

		Largest	Smallest
		Exponent	Exponent
		and then	start with 0
		count down	and count up
()() [*] (†)) * *
Pascal	1st Term	Last Term	
Triangle	in the	in the	
Number	binomial	binomial	

4. Expand $(2x + 3)^4$

You Try:

1. Expand (x + 1)⁴

2. Expand (x + 3)⁵

3. Expand $(3x + 5)^3$

To expand binomials representing differences, rather than sums, the binomial coefficients will remain the same but the signs will alternate beginning with positive, then negative, then positive, and so on. Compare the result.

$$(x+2)^{3} = (1)(x^{3})(2^{0}) + (3)(x^{2})(2^{1}) + (3)(x^{1})(2^{2}) + (1)(x^{0})(2^{3}) = x^{3} + 6x^{2} + 12x + 8$$
$$(x-2)^{3} = (1)(x^{3})(2^{0}) - (3)(x^{2})(2^{1}) + (3)(x^{1})(2^{2}) - (1)(x^{0})(2^{3}) = x^{3} - 6x^{2} + 12x - 8$$

What do you notice?? _____

Ex. Expand $(x - 2)^4$

Ex. Expand $(4x - 3)^3$

You Try:

1. Expand $(x - 5)^3$

2. Expand $(x - 2)^5$

3. Expand $(2x - 7)^4$

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6... 5... 4... 3... 2... x^2 would be the 5th term.

You Try:

1. What is 5^{th} term of $(x + 4)^9$?

2. What is just the coefficient of the 3^{rd} term of $(3t + 5)^8$?

More Practice:

1.	(x + 3) ⁴
2.	$(x - 4)^3$
3.	(x + 2) ³
4.	(x - 5) ⁴
5.	$(x - 1)^7$
6.	(3x + 2) ⁶

- 7. (5x + 3)³
- 8. Find the 5^{th} term in the expansion of $(2x + 4)^{11}$.
- 9. Find the 4^{th} term in the expansion of $(3x 2)^{10}$.
- 10. Find just the <u>coefficient</u> of the 2^{nd} term in the expansion of $(x 3)^7$.

Lesson 2-3: Synthetic and Long Division

Learning Target: R2:

Polynomial Long Division

Ex. $(2x^2 - 3x + 1) \div (x + 5)$

Ex. $(3x^2 - x + 4) \div (x - 1)$

Ex. $(x^4 - 3x^3 - 2x + 1) \div (x^2 + 1)$

You Try:

1. $(x^2 - 2x + 6) \div (x + 2)$

2. $(x^2 - 5x - 7) \div (x - 8)$

More Practice with Long Division:

1.
$$(x^2 + 3x - 6) \div (x + 1)$$

2. $(x^2 + x - 3) \div (x - 2)$

3. $(x^4 + 5x^3 + 3x^2 - 8x + 3) \div (x + 3)$

4. $(4x^4 + 3x^2 + 2x - 2) \div (2x - 1)$

5. $(2x^3 + 3x^2 + 4x - 5) \div (x - 2)$

6. $(6x^4 + 3x^3 - 5x^2 + 2x + 4) \div (3x + 1)$

Polynomial Synthetic Division

Ex. $(x^2 - 4x + 3) \div (x - 1)$ Ex. $(2x^2 - 5x - 3) \div (x - 3)$

Ex. $(x^3 - 10x - 24) \div (x - 3)$

Ex. $(2x^4 - 5x^3 - 7x - 6) \div (x - 3)$

You Try:

1. $(x^2 + 2x - 15) \div (x + 4)$

2. $(4x^4 + 3x^2 + 2x - 2) \div (x - 1)$

More Practice with Synthetic Division:

1.
$$(x^2 + 7x + 4) \div (x + 3)$$

2. $(x^2 + 2x + 1) \div (x + 1)$

3. $(3x^2 + 10x + 8) \div (x + 2)$

4. $(x^4 + 7x^3 + 17x^2 + 13x - 6) \div (x + 3)$

5. $(x^4 + 3x^3 + 2x^2 + x - 6) \div (x - 2)$

6. $(3x^4 + 10x^3 - 6x^2 + 5x - 7) \div (x + 4)$

Review of Polynomial Operations and the Binomial Theorem:

Add or Subtract the following (Circle your final answer):

1.
$$(10x^2+3) + (x^2+6x+8)$$

2. $(-5x^2+1) - (6x^2-2)$
3. $(7x^3+3x+1) + (2x^2-5x-9)$

Multiply the following (Circle your final answer):

4.
$$(x + 7) (4x + 5)$$
 5. $(2x - 5)(2x^2 - 3x + 4)$ 6. $(3x + 4)^3$

Use Pascal's triangle for the following:

7. Expand $(x - 2)^3$

8. Expand $(2x + 3)^4$

9. Find the third term of $(x - 3)^5$.

Divide the following using either Polynomial Long Division or Polynomial Synthetic Division (Circle your final answer):

10.
$$(x^2 + 7x + 8) \div (x + 2)$$

11. $(7x^2 + 5x - 12) \div (2x - 5)$
12. $(x^3 + 12x^2 + 48x + 64) \div (x + 4)$

13.
$$(6x^2 - 13x - 63) \div (2x - 9)$$
 14. $(3x^3 - 6x - 4) \div (x + 3)$ 15. $(x^3 - x^2 - 7x - 2) \div (x + 1)$

Lesson 2-4: Composition of Functions

Learning Target: R3:			
Learning Target: K2:			
Learning Target: R4:			
Evaluate each function:			
1. $f(4) = 3x - 5$	2. $g(-7) = x^2 + 9$	3. $h(x + 3) = 2x - 5$	

6. p(2x + 3) = 5x + 4

Ex. Use the functions f(x) = 4x - 2 and g(x) = 3x + 1 to answer the following questions:

5. h(3) = 2x - 5

7. f(5) = 8. g(-2) = 9. f(x) + g(x) =

10.
$$f(x) - g(x) =$$
 11. $g(x) - f(x) =$ 12. $f(x) \cdot g(x) =$

You Try: Use the functions f(x) = 3x - 4 and g(x) = 4x + 5 to answer the following questions:

13. f(-7) = 14. g(3) = 15. f(x) + g(x) =

16.
$$f(x) - g(x) =$$
 17. $g(x) - f(x) =$ 18. $f(x) \cdot g(x) =$

b. g(f(x)) =

Ex. Find f(g(x)) and g(f(x)). Simplify your answer:

4. $f(2) = x^2 - 4x + 1$

19. f(x) = 2x g(x) = x + 5a. f(g(x)) =

20.
$$f(x) = x^2 + 2$$
 $g(x) = x - 1$
a. $f(g(x)) =$ b. $g(f(x)) =$

21.
$$f(x) = 3x$$
 $g(x) = 4x + 9$

a. f(g(x)) = b. g(f(x)) =

You Try:

22. $f(x) = x^2 - 5$ g(x) = 2xa. f(g(x)) = b. g(f(x)) =

Use the functions f(x) = 3x - 5 and g(x) = 2x + 4 to answer the following questions:

23. f(4) = 24. g(5) = 25. f(x) + g(x) =

26. f(x) - g(x) = 27. g(x) - f(x) = 28. $f(x) \cdot g(x) =$

29. $f(g(x)) =$	30. $g(f(x)) =$	31. $f(f(x)) =$
29.1(g(X)) =	50. g(1(x)) =	$J_{1} = I(I(X)) =$

Lesson 2-5: Inverse of Functions

Inverse Functions – Finding an Inverse

Learning Target: R5: _____

Examples – Find the inverse functions of the following:

STEP 1: Swap x and y (remember f(x), g(x), etc are the same thing as y) STEP 2: Solve for y STEP 3: Rewrite in inverse notation ($f^{-1}(x)$, $g^{-1}(x)$, etc)

1. $f(x) = 5x + 3$	2. $g(x) = \frac{3x+1}{5}$	3. $h(x) = \frac{x}{2} - 6$	4. $g(x) = 2x^3 + 6$

You Practice:

Find the inverse relations for the following:

1. $g(x) = 5x$	2. $f(x) = 2x - 1$	3. $h(x) = -\frac{2}{3}x + 6$
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4. $f(x) = -2x + 5$	5. $g(x) = 3x - 3$	6. $h(x) = \frac{1}{2}x + 6$
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7.
$$g(x) = -\frac{4}{5}x + 11$$

8. $f(x) = 11x - 5$
9. $f(x) = -12x + 7$
10. $f(x) = 3x - \frac{1}{4}$
11. $g(x) = 8x - 13$
12. $h(x) = -\frac{3}{7}x + \frac{5}{7}$

11. g(x) = 8x - 13

13.
$$f(x) = x^7 + 4$$
 14. $f(x) = 3x^3$ 15. $f(x) = x^3 + 2$

16.
$$f(x) = \frac{3}{5}x^3 - 9$$
 17. $f(x) = x^4 - \frac{1}{2}$ 18. $f(x) = \frac{1}{6}x^5 + \frac{2}{3}$

Inverse Functions – Verifying Inverses

Learning Target: R6: ____

Example: Verify whether or not the following are inverse functions:

STEP 1: Use composition of functions. For example, find f(g(x)) and g(f(x)). STEP 2: Compare the solutions. If <u>both</u> solutions is equal to x the functions are inverses of each other.

Ex 1: f(x) = 4x and $g(x) = \frac{x}{4}$ Ex 2: f(x) = 2x + 1 and $g(x) = \frac{x-1}{2}$

Ex 3:
$$f(x) = \frac{x}{2} - 3$$
 and $g(x) = 2x + 3$
Ex 4: $f(x) = 2x^3 - 1$ and $g(x) = \frac{3}{\sqrt{\frac{x+1}{2}}}$

You Try:

1.
$$f(x) = 2x - 4$$
 and $g(x) = \frac{1}{2}x + 2$
2. $f(x) = 8x^3$ and $g(x) = \frac{x^{1/3}}{2}$

3. f(x) = 6x + 3 and $g(x) = \frac{1}{6}x - \frac{1}{2}$ 4. f(x) = x + 7 and g(x) = x - 7

5.
$$f(x) = 3x - 1$$
 and $g(x) = \frac{1}{3}x + \frac{1}{3}$
6. $f(x) = \frac{1}{2}x + 1$ and $g(x) = 2x - 2$

7.
$$f(x) = -2x + 4$$
 and $g(x) = -\frac{1}{2}x + 2$
8. $f(x) = 3x^3 + 1$ and $g(x) = \left(\frac{x-1}{3}\right)^{1/3}$

Inverse Functions – Inverses using graphs and tables

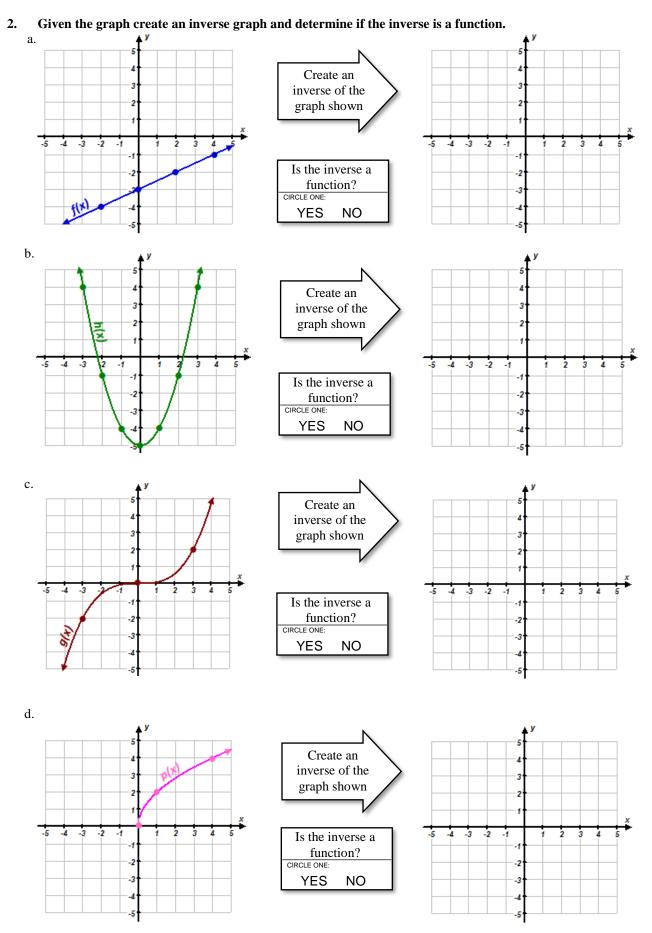
Learning Target: K3: _____

1. Which function is the inverse of the first table shown?

X	f(x)
- 2	0
-1	2
0	- 2
2	-1

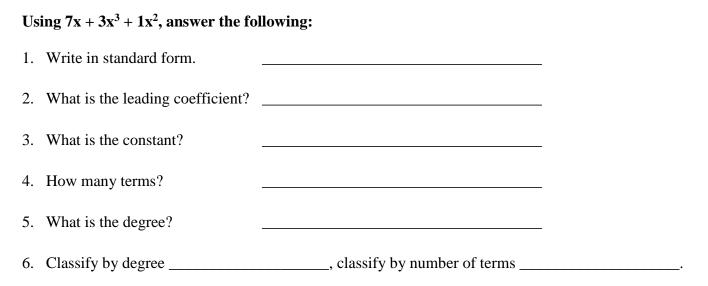
x	g(x)
0	- 2
2	-1
- 2	0
-1	2

X	h(x)
- 2	-1
-1	- 2
0	2
2	0



23

UNIT 2 STUDY GUIDE



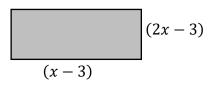
Add, Subtract, or Multiply Polynomials

7. $(x^2 - 4x + 12) + (x - 5)$

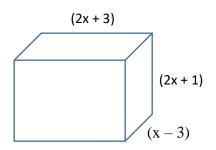
8. $(x^2 + 5x - 6) - (x^2 - 3x + 4)$

9. (2x+3)(x-2)

10. Find the area of the rectangle:



11. Find the volume (no parentheses in answer).



Polynomials Division:

12.
$$(3x^3 + 10x^2 + 13x + 10) \div (x + 2)$$
 13. $(3x^2 + 4x - 6) \div (x - 3)$

14.
$$(3x^3 + 10x^2 + 13x + 10) \div (x + 2)$$
 15. $(6x^2 + 9x - 15) \div (2x + 5)$

Use Pascal's triangle for the following:

16. Expand $(x - 2)^3$ 17. Find the third term of $(x - 3)^5$.

18. Expand
$$(2x + 3)^4$$
 19. $(x - 13)^1 =$

Use Composition of Functions:

20.
$$f(x) = x^2$$
 and $g(x) = x^2 + 5$
a. Find $f(x) + g(x)$
b. Find $f(x) - g(x)$
c. Find $f(4)$

d. Find $f(x) \cdot g(x)$ e. Find f(g(x)) f. Find g(f(x))

Find the Inverse:

21.
$$f(x) = 2x^3 + 1$$
, find $f^{-1}(x)$

22. f(x) = 6x - 2, find $f^{-1}(x)$

Verify "if" the following functions are inverses.

23. $f(x) = x^2 + 2$ $g(x) = \sqrt{x - 2}$

24.
$$f(x) = 2x - 7$$
 $g(x) = \frac{x - 7}{2}$

25. Create a Table that reflects the inverse of the given table.

x	h(x)
- 2	-1
-1	- 2
0	2
2	0

Х	h'(x)

UNIT 2 Reflection	Name:	Period:	
Do you feel like you stay on task during class?			
Do you feel like you are struggling with the	content?		
What do you think is your biggest distraction	in class?		
What about this unit did you find to be the ea	siest?		
What about this unit did you find to be the most difficult?			
How can I help you be more successful?			