NAME: ______________________________ PERIOD: _______

Algebra II
UNIT 3

Polynomial Functions

WHAT ARE YOU LEARNING?

Henry County Graduate Learner Outcomes:

- As a Henry County graduate, I will be able to create, interpret, use, and analyze patterns of algebraic structures to make sense of problems.
- As a Henry County graduate, I will be able to use functions to interpret and analyze a variety of contexts.

Georgia Standards of Excellence:

Lesson 3-1 – Reviewing Factoring and the Quadratic Formula

MGSE9-12.A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients, in context.

Lesson 3-2 – Rational Root Theorem and Fundamental Theorem of Algebra

MGSE9-12.N.CN.9 Use the Fundamental Theorem of Algebra to find all roots of a polynomial equation.

MGSE9-12.A.APR.2 Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).

Lesson 3-3 – Polynomial Symmetry

MGSE9-12.F.IF.4 Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums;
symmetries; and end behavior.

Lesson 3-4 – Characteristics of Polynomial Functions

MGSE9-12.A.APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

MGSE9-12.F.IF.7 Graph functions expressed algebraically and show key features of the graph both by hand and by using technology.

MGSE9-12.F.IF.7c Graph polynomial
functions, identifying zeros when suitable factorizations are available, and showing end behavior.

WHY ARE YOU LEARNING THIS?

Level 3 Performance Task:

Create a video game scene representing the various characteristics of a polynomial function. More details to follow.

WHAT IS YOUR GOAL FOR THIS UNIT?

Unit Goal:

I scored a __________ on my pretest.

My goal is to score a __________ or higher on the end of unit test.

To achieve this goal I will

____

____

HOW WILL YOU KNOW WHEN YOU'VE MASTERED THIS? SHOW ME THE EVIDENCE!

Data Analysis: **Pre-Test Score ________________** **Post-Test Score ________________**

Learning Targets:	**Pre-Test Score**	**Quiz Score**	**Post-Test Score**
K1: I can identify roots by factoring and/or using the quadratic formula. (A.SSE.1a)			
R1: I can apply the Remainder Theorem (Rational Root Theorem) and the Fundamental Theorem of Algebra to identify zeros of a polynomial function. (N.CN.9 ,A.APR.2)			
K2: I can identify odd, even, or neither symmetry graphically or algebraically.			
R2: I can sketch a graph (by hand and using technology) showing the key characteristics of a polynomial function. (F.IF.4, A.APR.3, F.IF.7, F.IF.7c)			

LEARNING ACTIVITIES

Lesson 3-1 – Review of Factoring and the Quadratic Formula

K1: I can identify roots by factoring and/or using the quadratic formula. (A.SSE.1a)

______ Complete guided notes with teacher **OR** Watch video lesson and take notes
______ Complete 3-1 practice

Lesson 3-2 – Rational Root Theorem and the Fundamental Theorem of Algebra

R1: I can apply the Remainder Theorem (Rational Root Theorem) and the Fundamental Theorem of Algebra to identify zeros of a polynomial function.
(N.CN.9 ,A.APR.2)

______ Complete guided notes with teacher **OR** Watch video lesson and take notes
______ Complete 3-2 practice

Quiz 3-1 to 3-2

Lesson 3-3 – Polynomial Symmetry

K2: I can identify odd, even, or neither symmetry graphically or algebraically.

______ Complete guided notes with teacher **OR** Watch video lesson and take notes
______ Complete 3-3 practice

Lesson 3-4 – Characteristics of Polynomial Functions

R2: I can sketch a graph (by hand and using technology) showing the key characteristics of a polynomial function. (F.IF.4, A.APR.3, F.IF.7, F.IF.7c)

______ Complete guided notes with teacher **OR** Watch video lesson and take notes
______ Complete 3-4 practice

Quiz 3-3 to 3-4

UNIT 3 Assessments

______ Complete Unit 3 Performance Task
______ Complete Unit 3 Review Guide
______ Complete Unit 3 Test
______ Complete Unit 3 Reflection

Lesson 3-1: Review of Factoring and the Quadratic Formula

Learning Target: K1: I can identify roots by factoring and/or using the quadratic formula.

FACTORING REVIEW: Factoring is used to solve quadratics of the form $ax^2 + bx + c = 0$ when the roots are rational. Find the roots of the following quadratic functions:

a. $f(x) = x^2 - 5x - 14$

b. $f(x) = x^2 - 64$

c. $f(x) = 6x^2 + 7x - 3$

d. $f(x) = 3x^2 + x - 2$

You Try:

1. $f(x) = x^2 + 5x + 4$
2. $f(x) = 2x^2 - 9x + 10$
3. $f(x) = x^2 + 13x + 40$
4. $f(x) = 3x^2 + 32x - 11$
5. $f(x) = x^2 - 5x - 6$
6. $f(x) = x^2 - 16$
7. $f(x) = x^2 - 4x + 3$
8. $f(x) = 5x^2 + 17x - 12$
9. $f(x) = x^2 - 14x - 72$
10. $f(x) = x^2 - 9x + 14$
11. $f(x) = x^2 + 3x - 10$
12. $f(x) = x^2 - 8x + 12$
13. $f(x) = 2x^2 - 15x - 8$
14. $f(x) = x^2 + 6x - 27$
15. $f(x) = 4x^2 - 9$

QUADRATIC FORMULA REVIEW:

Another option for solving a quadratic whether it is factorable but particularly when it is not is to use the quadratic formula. Remember, a quadratic equation written in $ax^2 + bx + c = 0$ has solution(s)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Also remember that $b^2 - 4ac$ is the discriminant and gives us the ability to determine the nature of the roots.

$$b^2 - 4ac \begin{cases} > 0 & 2 \quad \textit{real roots} \\ = 0 & 1 \quad \textit{real root} \\ < 0 & 0 \quad \textit{real roots (imaginary)} \end{cases}$$

Find the roots for each of the following. Also, describe the number and nature of these roots.

a. $f(x) = 4x^2 - 2x + 9$

b. $f(x) = 3x^2 + 4x - 8$

c. $f(x) = x^2 - 5x + 9$

You Try:

1. $f(x) = x^2 – 2x + 3$

2. $f(x) = x^2 + 4x + 4$

3. $f(x) = 5x^2 – 2x + 4$

4. $f(x) = 3x^2 + 32x – 11$

5. $f(x) = x^2 – 5x – 6$

6. $f(x) = 4x^2 – 12x + 9$

7. $f(x) = -x^2 – 3x – 5$

8. $f(x) = x^2 + 5x + 2$

9. $f(x) = 2x^2 + 5x – 1$

Lesson 3-2: Rational Root Theorem and Fundamental Theorem of Algebra

Learning Target: R1: I can apply the Remainder Theorem (Rational Root Theorem) and the Fundamental Theorem of Algebra to identify zeros of a polynomial function.

RATIONAL ROOT THEOREM:

The Rational Root Theorem states that any rational solutions to a polynomial will be in the form of $\frac{p}{q}$ where p is a factor of the constant term of the polynomial (the term that does not show a variable) and q is a factor of the leading coefficient. This is actually much simpler than it appears at first glance.

Example: Let us consider the polynomial $f(x) = x^3 - 5x^2 - 4x + 20$

Identify p : ____

Identify q : ____

Identify all possible combinations of $\frac{p}{q}$: ______________________________

Find the roots using Synthetic Division and Quadratic Formula (or Factoring):

Identify the roots: ______________________________________

Identify the factors: ______________________________________

Use the Quadratic Formula or Factoring

a. $f(x) = x^3 + 2x^2 - 5x - 6$

Identify p : ______________________________

Identify q : ______________________________

Identify all possible combinations of $\frac{p}{q}$: ______________________________

Find the roots using Synthetic Division and Quadratic Formula (or Factoring):

Identify the roots: ______________________________

Identify the factors: ______________________________

b. $f(x) = 4x^3 - 7x + 3$

Identify p : ______________________________

Identify q : ______________________________

Identify all possible combinations of $\frac{p}{q}$: ______________________________

Find the roots using Synthetic Division and Quadratic Formula (or Factoring):

Identify the roots: ______________________________

Identify the factors: ______________________________

c. $f(x) = 3x^3 + 25x^2 + 56x + 16$

Identify p : ______________________________

Identify q : ______________________________

Identify all possible combinations of $\frac{p}{q}$: ______________________________

Find the roots using Synthetic Division and Quadratic Formula (or Factoring):

Identify the roots: ______________________________

Identify the factors: ______________________________

d. $f(x) = x^4 + 2x^3 - 9x^2 - 2x + 8$

Identify p : ______________________________

Identify q : ______________________________

Identify all possible combinations of $\frac{p}{q}$: ______________________________

Find the roots using Synthetic Division and Quadratic Formula (or Factoring):

Identify the roots: ______________________________

Identify the factors: ______________________________

e. $f(x) = x^4 - 11x^3 - 13x^2 + 11x + 12$

Identify p : ______________________________

Identify q : ______________________________

Identify all possible combinations of $\frac{p}{q}$: ______________________________

Find the roots using Synthetic Division and Quadratic Formula (or Factoring):

Identify the roots: ______________________________

Identify the factors: ______________________________

f. $f(x) = x^5 - 12x^4 + 49x^3 - 90x^2 + 76x - 24$

Identify p : ______________________________

Identify q : ______________________________

Identify all possible combinations of $\frac{p}{q}$: ______________________________

Find the roots using Synthetic Division and Quadratic Formula (or Factoring):

Identify the roots: ______________________________

Identify the factors: ______________________________

g. $f(x) = x^4 - 8x^3 + 24x^2 - 32x + 16$

Identify p : ____

Identify q : ____

Identify all possible combinations of $\frac{p}{q}$: ____________________________

Find the roots using Synthetic Division and Quadratic Formula (or Factoring):

Identify the roots: ____

Identify the factors: ____

h. $x^5 + x^4 - 31x^3 + 11x^2 + 114x + 72$

Identify p : ____

Identify q : ____

Identify all possible combinations of $\frac{p}{q}$: ____________________________

Find the roots using Synthetic Division and Quadratic Formula (or Factoring):

Identify the roots: ____

Identify the factors: ____

Lesson 3-3: Polynomial Symmetry (Odd, Even, and Neither Functions)

Learning Target: K2: I can identify odd, even, or neither symmetry graphically or algebraically.

There are 2 ways to tell if a function is odd or even… Algebraically and Graphically.

Algebraically:

BEWARE OF CONSTANTS

All constants really have a x^0

x^0 is EVEN!!!

A function has __________ symmetry if all variable exponents are even.

A function has __________ symmetry if all variable exponents are odd.

A function has __________ symmetry if the exponents are a mixture of odd and even.

Examples:

1. $f(x) = x^3 - x$

2. $f(x) = x^2 + 1$

3. $f(x) = x^2 + 6x - 4$

4. $f(x) = 2x^4 - 3$

5. $f(x) = -x^3$

6. $f(x) = x^2 + 4$

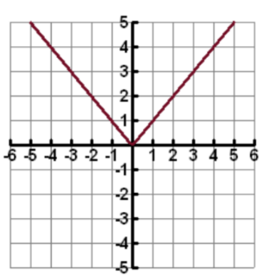
Graphically:

A function has __________ symmetry if the graph reflects across the y-axis.
(meaning… you can fold it hotdog style and it would match up with the y-axis in the middle)

A function has __________ symmetry if the graph has 180^0 rotational symmetry about the origin.
(meaning… you can turn it upside down and it will look exactly the same… it also must go through the origin).

Examples:

1.



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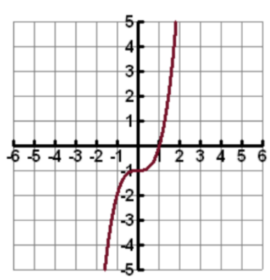

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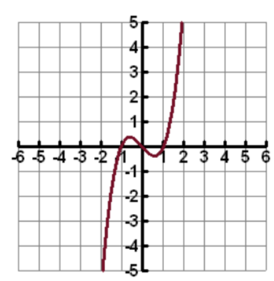
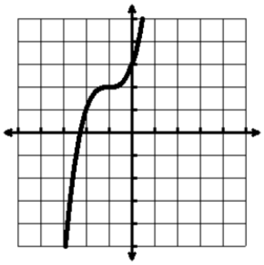
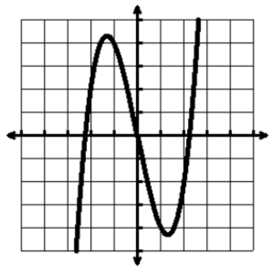
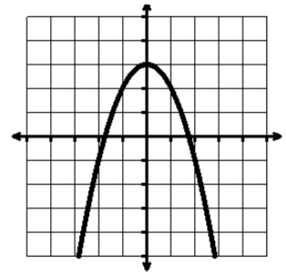


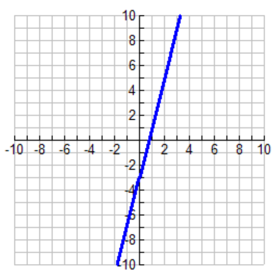
4. 

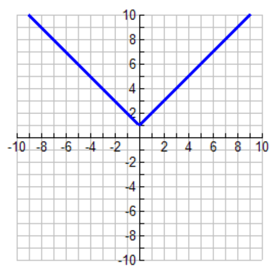
5. 

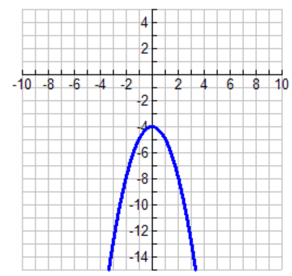
6. 

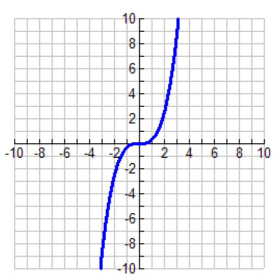
You Practice:

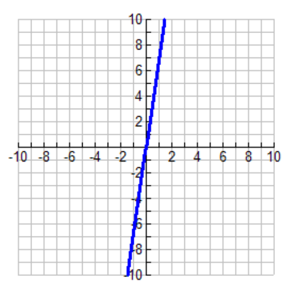
Determine whether the following functions have even, odd, or neither symmetry.

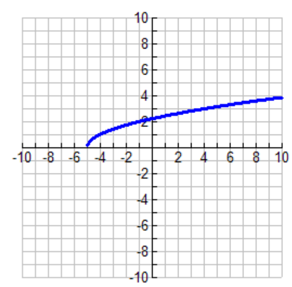
1. 

2. 

3. 

4. 

5. 

6. 

7. $f(x)= 3x^2$

8. $f(x) = x^3 – 2$

9. $f(x)= 3x + 4$

10. $f(x)= x^2 – 5$

11. $f(x)= 10x + 5$

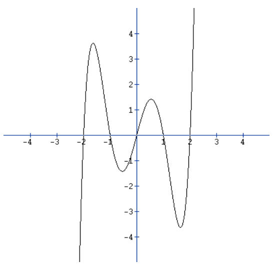
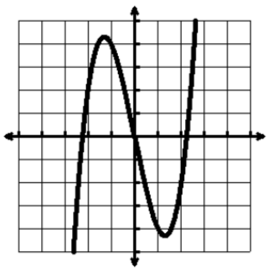
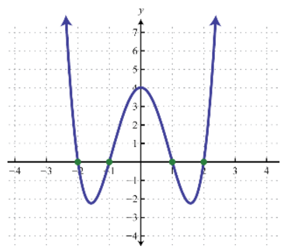
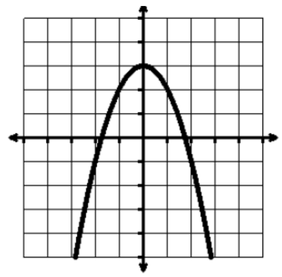
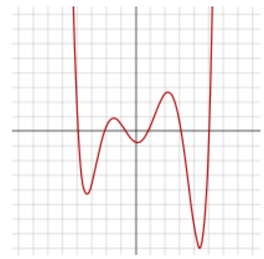
12. $f(x)= 2(x+1)^2$ Hint: Multiply 1st

Lesson 3-4: Characteristics of Polynomial Functions

Learning Target: R2: I can sketch a graph (by hand and using technology) showing the key characteristics of a polynomial function.

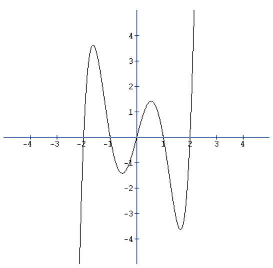
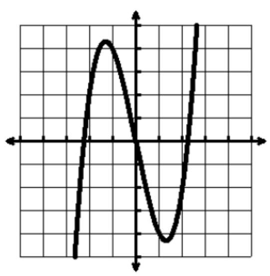
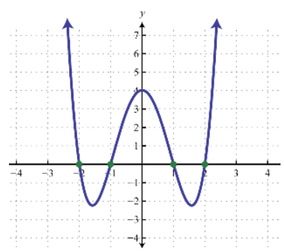
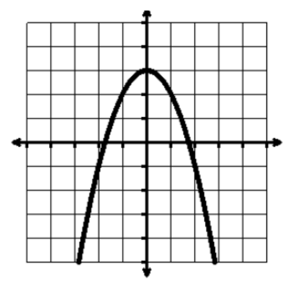
Odd or Even Function:

- If the function has a leading degree that is odd, it will be an odd function, meaning the end behavior will go in opposite directions on each side.
- If the function has a leading degree that is even, it will be an even function, meaning the end behavior will go in the same direction on each side.

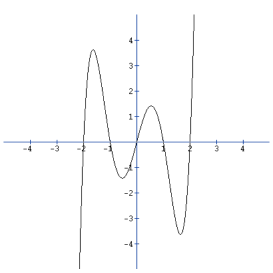
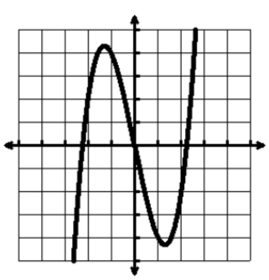
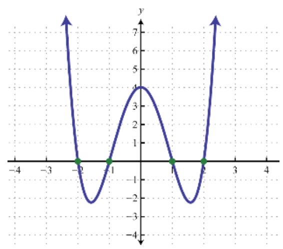
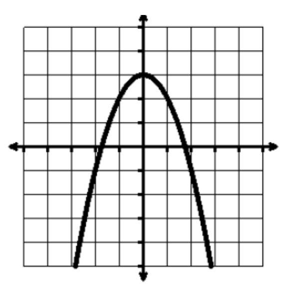
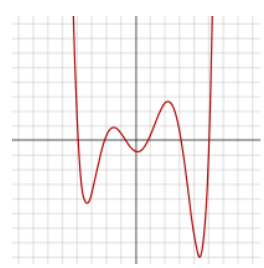
Odd / Even   Odd / Even   Odd / Even   Odd / Even   Odd / Even

Zeros (x-intercepts): Where the graph crosses the x-axis.
Y-intercept: Where the graph crosses the y-axis. This will also be the constant when in standard form.

Zeros:	Zeros:	Zeros:	Zeros:
Y-Int:	Y-Int:	Y-Int:	Y-Int:

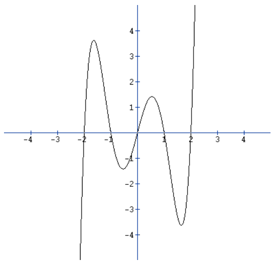
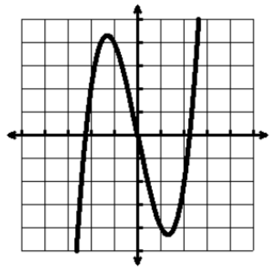
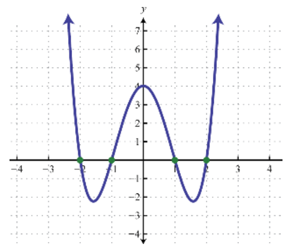
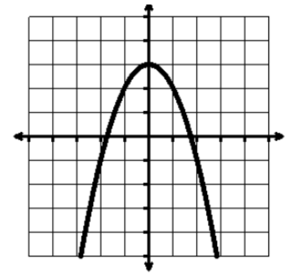
Domain: Input values – how far to the left and right a graph can go. Always small to big.
Range: Output values – how far up and down a graph can go. Always small to big.

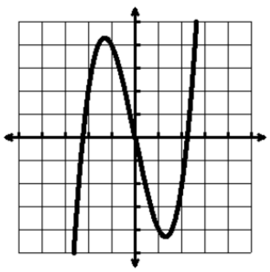
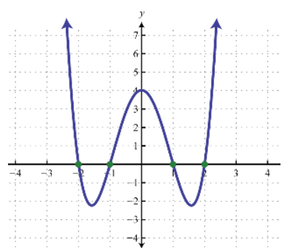
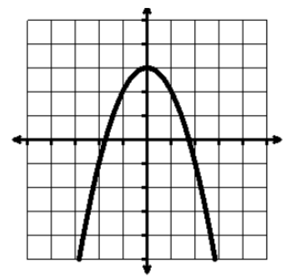
D:	D:	D:	D:	D:
R:	R:	R:	R:	R:

Minimum and Maximum

- Absolute Minimum and Maximum will be how low or high a graph can go if there is an exact point.
- Relative Minimum and Maximum will be how low or high a graph can go at its peaks. Also called an extrema.

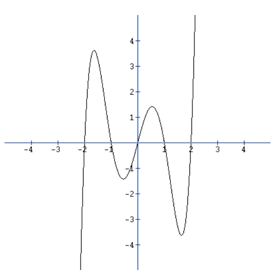
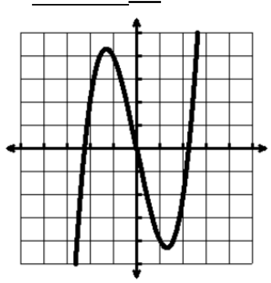
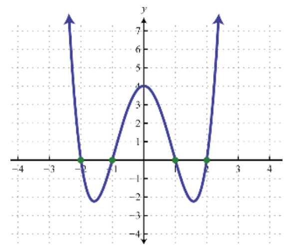
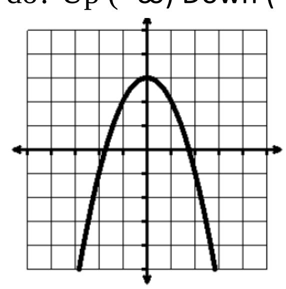





Interval of Increase and Decrease: You will read the graph from left to right and determine the intervals at which the graph increases and decreases as you move to the right. Always use the x-values.

End Behavior: You will look at the graph from the left and the right side and determine if the graph is going up or down as you move the left and the right.

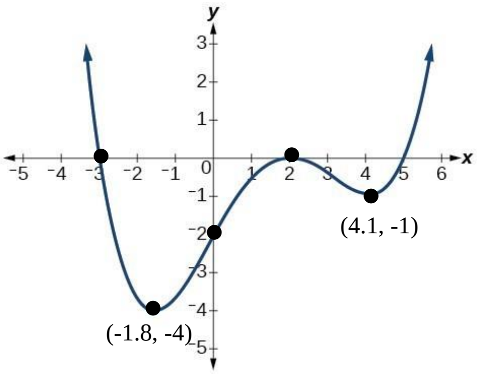
- As $x \to -\infty$, $f(x) \to$ _________ As x goes to the left, what does y do? Up ($+\infty$) Down ($-\infty$)
- As $x \to +\infty$, $f(x) \to$ _________ As x goes to the right, what does y do? Up ($+\infty$) Down ($-\infty$)

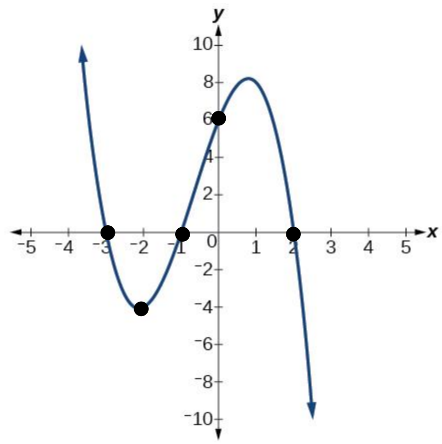





As $x \to -\infty$, $f(x) \to$ _____ As $x \to -\infty$, $f(x) \to$ _____ As $x \to -\infty$, $f(x) \to$ _____ As $x \to -\infty$, $f(x) \to$ _____

As $x \to +\infty$, $f(x) \to$ _____ As $x \to +\infty$, $f(x) \to$ _____ As $x \to +\infty$, $f(x) \to$ _____ As $x \to +\infty$, $f(x) \to$ _____

EXAMPLES:





a. Symmetry ________________

b. Odd / Even Function? _______

c. Zeros ________________

d. Y-Intercept ________________

e. Relative Max ________________

f. Absolute Min ________________

g. Domain ________________

h. Range ________________

i. Int of Increase ________________

j. Int of Decrease________________

k. End Behavior

As x → +∞, f(x) → ________

As x → –∞, f(x) → ________

a. Symmetry ________________

b. Odd / Even Function? _______

c. Zeros ________________

d. Y-Intercept ________________

e. Relative Max ________________

f. Relative Min ________________

g. Domain ________________

h. Range ________________

i. Int of Increase ________________

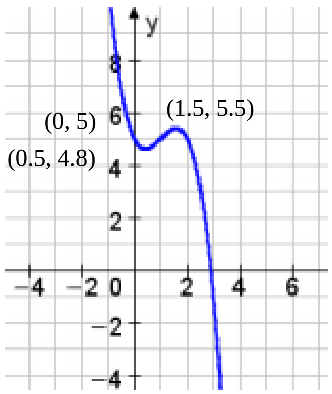
j. Int of Decrease ________________

k. End Behavior

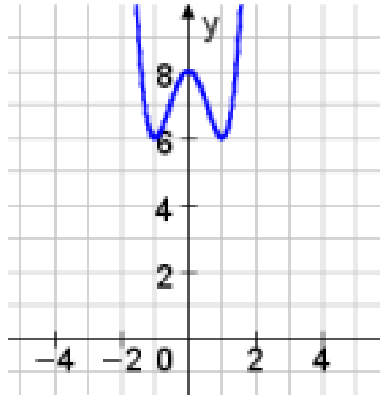
As x → +∞, f(x) → ________

As x → –∞, f(x) → ________

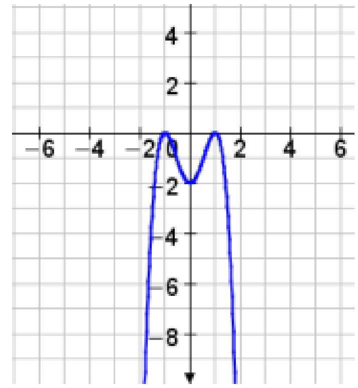
1.



2.



3.



a. Symmetry ______________

b. Odd / Even Function? ______

c. Zeros ______________

d. Y-Intercept ______________

e. Relative Max ______________

f. Relative Min ______________

g. Domain ______________

h. Range ______________

i. Int of Increase ______________

j. Int of Decrease ______________

k. End Behavior

As x → +∞, f(x) → ________

As x → −∞, f(x) → ________

a. Symmetry ______________

b. Odd / Even Function? ______

c. Zeros ______________

d. Y-Intercept ______________

e. Relative Max ______________

f. Absolute Min ______________

g. Domain ______________

h. Range ______________

i. Int of Increase ______________

j. Int of Decrease ______________

k. End Behavior

As x → +∞, f(x) → ________

As x → −∞, f(x) → ________

a. Symmetry ______________

b. Odd / Even Function? ______

c. Zeros ______________

d. Y-Intercept ______________

e. Absolute Max ______________

f. Relative Min ______________

g. Domain ______________

h. Range ______________

i. Int of Increase ______________

j. Int of Decrease ______________

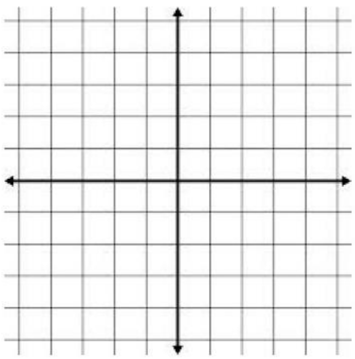
k. End Behavior

As x → +∞, f(x) → ________

As x → −∞, f(x) → ________

Graph the following polynomial functions and identify each function's characteristics:

4. $f(x) = x^3 - 3x^2 - x + 3$



a. Symmetry ______________

b. Odd / Even Function? _______

c. Zeros ______________

d. Y-Intercept ______________

e. Relative Max ______________

f. Relative Min ______________

g. Domain ______________

h. Range ______________

i. Int of Increase ______________

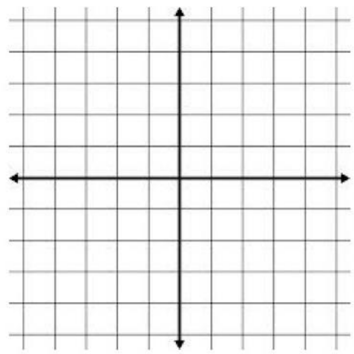
j. Int of Decrease ______________

k. End Behavior

As $x \to +\infty$, $f(x) \to$ ________

As $x \to -\infty$, $f(x) \to$ ________

5. $f(x) = -2x^4 + 3x^2 - 2$



a. Symmetry ______________

b. Odd / Even Function? _______

c. Zeros ______________

d. Y-Intercept ______________

e. Absolute Max ______________

f. Relative Min ______________

g. Domain ______________

h. Range ______________

i. Int of Increase ______________

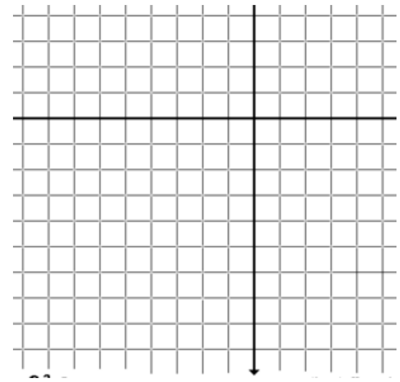
j. Int of Decrease ______________

k. End Behavior

As $x \to +\infty$, $f(x) \to$ ________

As $x \to -\infty$, $f(x) \to$ ________

6. $f(x) = x^3 + 5x^2 + 3x - 9$



a. Symmetry ______________

b. Odd / Even Function? _______

c. Zeros ______________

d. Y-Intercept ______________

e. Relative Max ______________

f. Relative Min ______________

g. Domain ______________

h. Range ______________

i. Int of Increase ______________

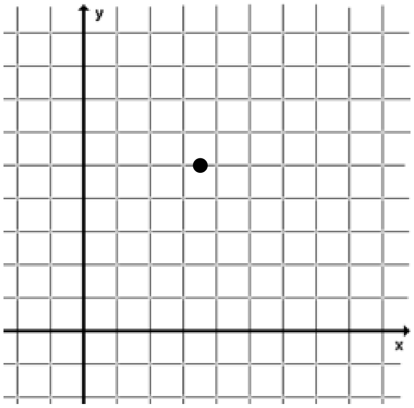
j. Int of Decrease ______________

k. End Behavior

As $x \to +\infty$, $f(x) \to$ ________

As $x \to -\infty$, $f(x) \to$ ________

7. $f(x) = x^4 - 14x^3 + 69x^2 - 140x + 100$



a. Symmetry ________________

b. Odd / Even Function? _______

c. Zeros ________________

d. Y-Intercept ________________

e. Relative Max ________________

f. Absolute Min ________________

g. Domain ________________

h. Range ________________

i. Int of Increase ________________

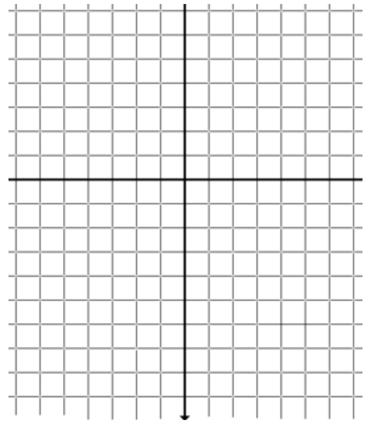
j. Int of Decrease ________________

k. End Behavior

As $x \to +\infty$, $f(x) \to$ ________

As $x \to -\infty$, $f(x) \to$ ________

8. $f(x) = x^2 - 2x - 8$



a. Symmetry ________________

b. Odd / Even Function? _______

c. Zeros ________________

d. Y-Intercept ________________

e. Max ________________

f. Absolute Min ________________

g. Domain ________________

h. Range ________________

i. Int of Increase ________________

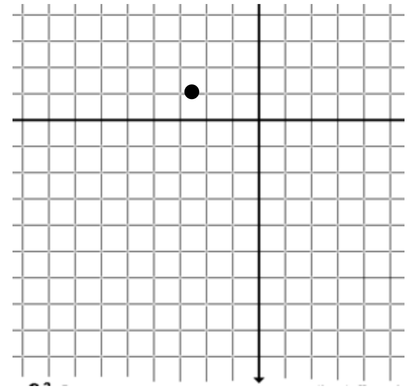
j. Int of Decrease ________________

k End Behavior

As $x \to +\infty$, $f(x) \to$ ________

As $x \to -\infty$, $f(x) \to$ ________

9 . $f(x) = x^3 + 4x^2 + x - 6$



a. Symmetry ________________

b. Odd / Even Function? _______

c. Zeros ________________

d. Y-Intercept ________________

e. Max ________________

f. Min ________________

g. Domain ________________

h. Range ________________

i. Int of Increase ________________

j. Int of Decrease ________________

k. End Behavior

As $x \to +\infty$, $f(x) \to$ ________

As $x \to -\infty$, $f(x) \to$ ________

UNIT 3 Reflection **Name: ______________________________ Period: _______**

Do you feel like you stay on task during class? ____

Do you feel like you are struggling with the content? ____

What do you think is your biggest distraction in class? ____

____

What about this unit did you find to be the easiest? ____

____

____

What about this unit did you find to be the most difficult? ____

____

____

How can I help you be more successful? ____

____

____

Give me feedback on the project. Did you like it? Did you not like it? Is there something I could do differently that would make it better?

____

____

____

____

Are you on track to pass this course? If not, what is your game plan? What are you going to do to help yourself master this course?

____

____