Hirsch

UNIT 3

Arc Length = $\frac{2\pi r\theta}{360}$ or $\frac{\text{angle}}{360} \cdot 2\pi r$

Sector Area = $\frac{\pi r^2\theta}{360}$ or $\frac{\text{angle}}{360} \cdot \pi r^2$

Area of a Circle: $A = \pi r^2$

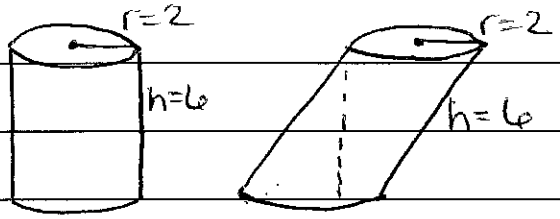
Circumference of a Circle: $C = 2\pi r$ or πd

VOLUME: Cylinder: $V = \pi r^2 h$

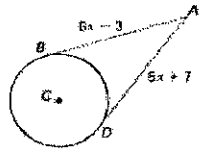
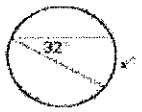
Pyramid: $V = \frac{1}{3}Bh$ ($B \rightarrow$ area of Base)

Cone: $V = \frac{1}{3}\pi r^2 h$

Sphere: $V = \frac{4}{3}\pi r^3$



same radius
same height
= same volume !!

EXAMPLE	RULE	WORKED OUT
	2 tangent segments are congruent when they are joined at a common exterior point. Tangent = Tangent	$6x-3=5x+7$ $-5x \quad -5x$ $x-3=7$ $+3 \quad +3$ $x=10$
	A central angle is equal to the intercepted arc. Central Angle = Arc	$x=123°$
	An inscribed angle is ½ the intercepted arc. Arc = 2(Angle) Angle = $\frac{\text{Arc}}{2}$	$x=32(2)$ $=64°$

Page 1

	If inscribed angles intercept the same arc, they are congruent. Angle = Angle	$5x-2=4x+9$ $-4x \quad -4x$ $x-2=9$ $+2 \quad +2$ $x=11$
	When a quadrilateral is inscribed in a circle, opposite angles are supplementary (add up to 180). Opp Angle + Opp Angle = 180	$y+87=180$ $-87 \quad -87$ $y=93°$ $2x+100=180$ $-100 \quad -100$ $\frac{2x}{2}=\frac{80}{2}$ $x=40$
	If a chord and a tangent intersect on the circle, the measure of the angle is ½ the measure of the intercepted arc. Arc = 2(Angle) Angle = $\frac{\text{Arc}}{2}$	$x=\frac{156}{2}=78°$ $y=180-78=102°$

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	If 2 chords intersect inside a circle, then the measure of each angle is ½ the sum of the measures of the arcs intercepted by the angle and its vertical angle. $\frac{\text{Arc}+\text{Arc}}{2}$ = Inside Angle	$\frac{110+30}{2}=x$ $\frac{140}{2}=x$ $70°=x$
	If a tangent and a secant, 2 tangents, or 2 secants intersect in the exterior of a circle, the measure of the angle formed is ½ the difference of the measures of the intercepted arcs. $\frac{\text{Big Arc}-\text{Little Arc}}{2}$ = Ext Angle	$\frac{171-85}{2}=x$ $\frac{86}{2}=x$ $43°=x$
	If 2 chords intersect in the interior of a circle then the product of each chord is congruent to the other. Chord 1 Part · Part = Chord 2 Part · Part	$3 \cdot 8 = 4 \cdot x$ $\frac{24}{4}=\frac{4x}{4}$ $x=6$

Page 3

	2 secant segments share the same exterior endpoint, then the product of the length of 1 secant segment and the length of its external segment = the product of the length of the other secant segment and the length of its external segment. Secant 1 Outside(whole) = Secant 2 Outside(whole)	$4(4+6)=5(x+5)$ $4(10)=5x+25$ $40=5x+25$ $-25 \quad -25$ $\frac{15}{5}=\frac{5x}{5}$ $x=3$
	A secant segment and a tangent segment share an exterior endpoint, then the product of the length of the secant segment & its external segment equals the square of the tangent segment length. Secant Outside(whole) = Tangent Outside(Outside)	$4(4+5)=x(x)$ $4(9)=x^2$ $36=x^2$ $\sqrt{36}=x$ $x=6$

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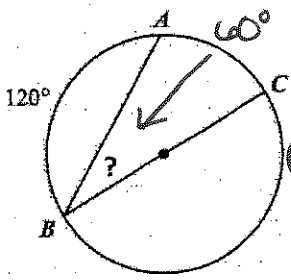
Unit 3 – EOC Practice

Name: _____

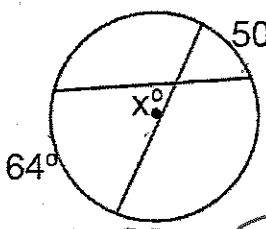
1. Solve for x or the missing measure:

a.



$\frac{60}{2}$ = 30°

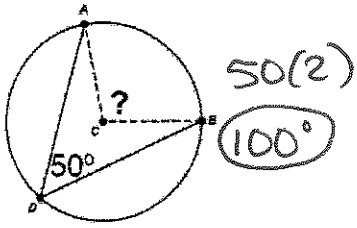
b.



$\frac{\text{arc}+\text{arc}}{2}$

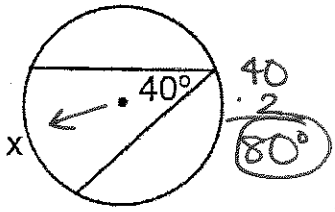
$\frac{64+50}{2}$ = 57°

c.



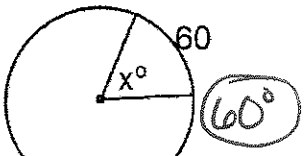
50(2) = 100°

d.



$\frac{40}{2}$ = 80°

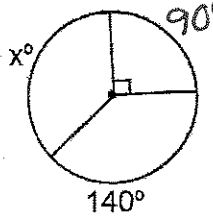
e.



60°

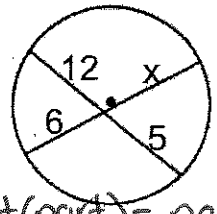
central = same

f.



360 − 140 − 90 = 130°

g.



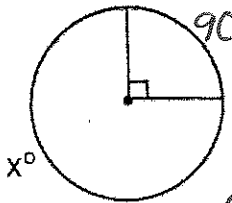
12(5) = 6(x)

$\frac{60}{6} = \frac{6x}{6}$

x = 10

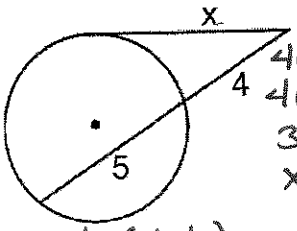
part(part) = part(part)

h.



360 − 90 = 270°

i.



4(4+5) = x(x)

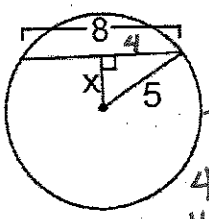
4(9) = x²

36 = x²

x = 6

outside (whole) = out (wh)

j.



chord is bisected.

- use Pyth. Th.

4² + x² = 5²

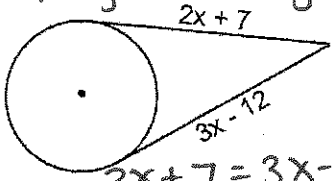
16 + x² = 25

−16 −16

x² = 9

x = 3

k. tangent = tangent

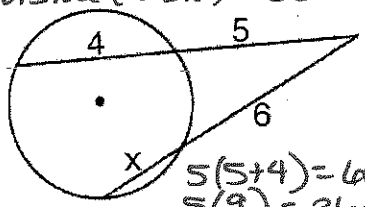


2x + 7 = 3x − 12

−2x +12 −2x +12

19 = x

l. outside (whole) = out (wh)



5(5+4) = 6(6+x)

5(9) = 36 + 6x

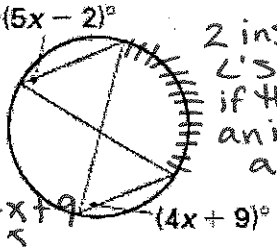
45 = 36 + 6x

−36 −36

$\frac{9}{6} = \frac{6x}{6}$

x = 1.5

m.



2 inscribed ∠'s are ≅ if they share an intercepted arc.

5x − 2 = 4x + 9

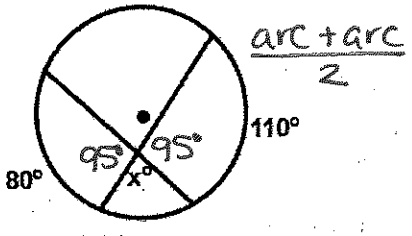
−4x −4x

x − 2 = 9

+2 +2

x = 11

n.



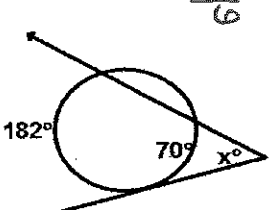
$\frac{\text{arc}+\text{arc}}{2}$

$\frac{80+110}{2}$ = 95

Now you have a linear pair.

180 − 95 = 85°

o.

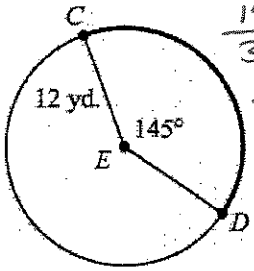


ext ∠ = $\frac{\text{arc}-\text{arc}}{2}$

$x = \frac{182-70}{2}$

x = 56°

2. Find the length of arc CD.



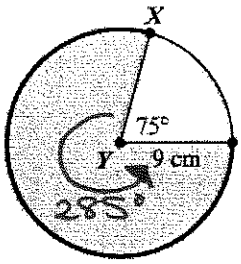
$\frac{145}{360} \cdot 2\pi r$

$\frac{145}{360} \cdot 2\pi(12)$

$= \frac{29\pi}{3}$ yd

or 30.369 yd

3. Find the Area of the Shaded Sector.



shaded is 285°

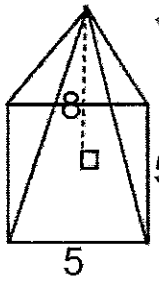
$\frac{285}{360} \cdot \pi r^2$

$\frac{285}{360} \cdot \pi(9)^2$

$= \frac{513\pi}{8}$ cm^3 or 201.455 cm^3

4. Find the Volume of the following:

a.



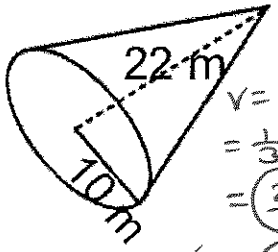
$V = \frac{1}{3}lwh$

$= \frac{1}{3}(5)(5)(8)$

$= \frac{200}{3}$

$= 66.667$ units^3

b.



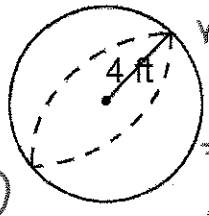
$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(10)^2(22)$

$= \frac{2200\pi}{3}$ m^3

or $= 2303.835$ m^3

c.



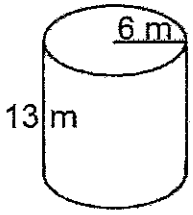
$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(4)^3$

$= \frac{256\pi}{3}$ ft^3

or $= 268.083$ ft^3

d.



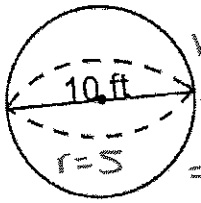
$V = \pi r^2 h$

$= \pi(6)^2(13)$

$= 468\pi$ m^3

or $= 1470.265$ m^3

e.



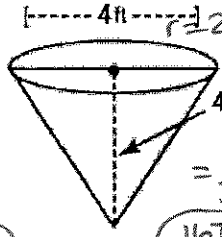
$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(5)^3$

$= \frac{500\pi}{3}$ ft^3

or $= 523.599$ ft^3

f.



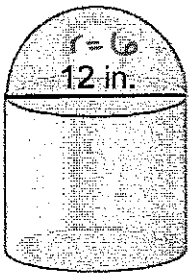
$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(2)^2(4)$

$= \frac{16\pi}{3}$ ft^3 or

$= 16.755$ ft^3

g.



sphere $V = \frac{4}{3}\pi r^3$ take ½

cylinder $V = \pi r^2 h$

sphere

$V = \frac{4}{3}\pi(6)^3$

$= 288\pi$

÷ by 2 $= 144\pi$

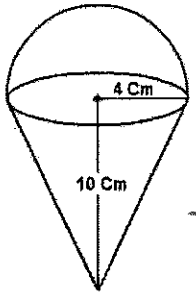
cylinder

$V = \pi(6)^2(13)$

$= 468\pi$

Together: $144\pi + 468\pi = 612\pi$ in^3 $= 1922.655$ in^3

h.



sphere: Take ½

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(4)^3$

$= \frac{256\pi}{3} \div 2 = \frac{128\pi}{3}$

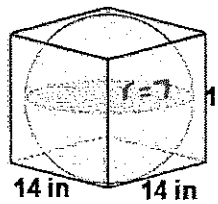
cone

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(4)^2(10)$

$= \frac{160\pi}{3}$

Together: $\frac{128\pi}{3} + \frac{160\pi}{3} = 96\pi$ cm^3 or 301.593 cm^3

i. Find the volume of the empty space.



sphere $V = \frac{4}{3}\pi r^3$

Box $V = lwh$

sphere

$V = \frac{4}{3}\pi(7)^3$

$= \frac{1372\pi}{3}$

Box

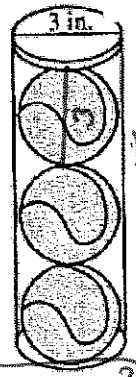
$V = 14(14)(14)$

$= 2744$

Box − Sphere $= 2744 - \frac{1372\pi}{3}$

empty space $= 1307.245$ in^3

j. Find the volume of the empty space.



sphere: $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1.5)^3$

$= 14.137$ per ball

$\times 3$

3 balls: 42.412 in^3

cylinder: $V = \pi r^2 h$

$= \pi(1.5)^2(9)$

$= 63.617$ in^3

$r = 1.5$

$h = 3(3) = 9$ in.

cylinder − spheres

$63.617 - 42.412 = 21.205$ in^3 empty space

5. **Circle P is dilated to form circle P'. Which statement is ALWAYS true?**

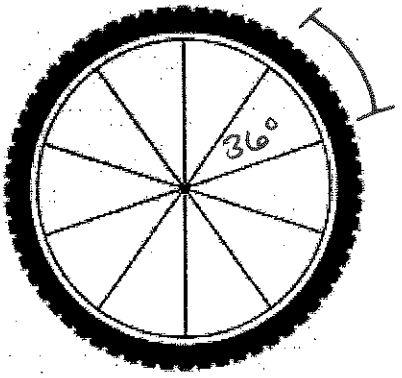
A. The radius of circle P is equal to the radius of circle P'.

B. The length of any chord in circle P is greater than the length of any chord in circle P'.

C. The diameter of circle P is greater than the diameter of circle P'.

D. The ratio of the diameter to the circumference is the same for both circles.

6. **The spokes of a bicycle wheel form 10 congruent central angles. The diameter of the circle formed by the outer edge of the wheel is 18 inches.**



$d = 18$ in
so $r = 9$ in.

What is the length, to the nearest 0.1 inch, of the outer edge of the wheel between two consecutive spokes?

$= \frac{36}{360} \cdot 2\pi r$

$= \frac{36}{360} \cdot 2\pi (9)$

$= 5.7$

A. 1.8 inches

B. 5.7 inches

C. 11.3 inches

D. 25.4 inches

7. **Jason constructed two cylinders using solid metal washers. The cylinders have the same height, but one of the cylinders is slanted as shown.**

Which statement is true about Jason's cylinders?

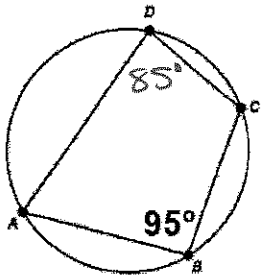
A. The cylinders have different volumes because they have different radii.

B. The cylinders have different volumes because they have different surface areas.

C. The cylinders have the same volume because each of the washers has the same height.

D. The cylinders have the same volume because they have the same cross-sectional area at every plane parallel to the bases.

8. Use the diagram to explain why $m\angle ADC = 85°$.



In an inscribed Quadrilateral, opposite angles are supplementary.
$180 - 95 = 85°$

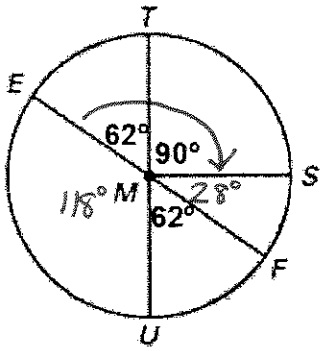
9. **What is the volume of a cylinder with a radius of 3 in. and a height of $\frac{9}{2}$ in.?**

A. $\frac{81}{2}\pi$ in.^{3} **C.** $\frac{27}{8}\pi$ in.^{3}

B. $\frac{27}{4}\pi$ in.^{3} **D.** $\frac{9}{4}\pi$ in.^{3}

$V = \pi r^2 h$
$= \pi(3)^2(\frac{9}{2})$
$= \frac{81\pi}{2}$

10. Use the diagram below to answer the following questions:

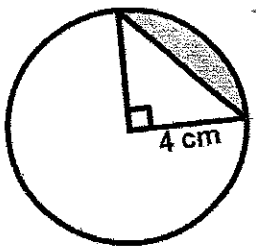


a. What is the m $\overset{\frown}{ET}$? $62°$

b. What is the m $\overset{\frown}{SF}$? $28°$

c. What is the m $\angle EMS$? $62 + 90 = 152°$

11. Find the area of the shaded region:

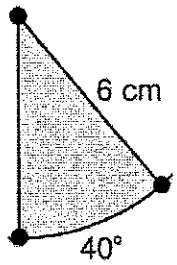


whole sector
$\frac{90}{360} \cdot \pi(4)^2$
$= 4\pi$

Triangle
$A = \frac{1}{2}bh = \frac{1}{2}(4)(4)$
$= 8$

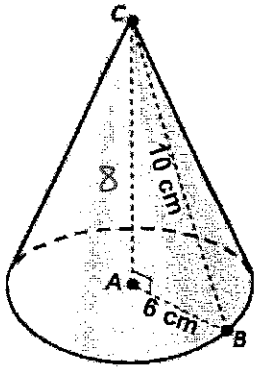
whole − Δ = $4\pi - 8 = 4.566\ cm^2$

12. Find the area of the following figure:



$A = \frac{40}{360} \cdot \pi r^2$
$= \frac{40}{360} \cdot \pi(6)^2$
$= 4\pi\ cm^2$
or $= 12.566\ cm^2$

13. Find the volume of the following cone.

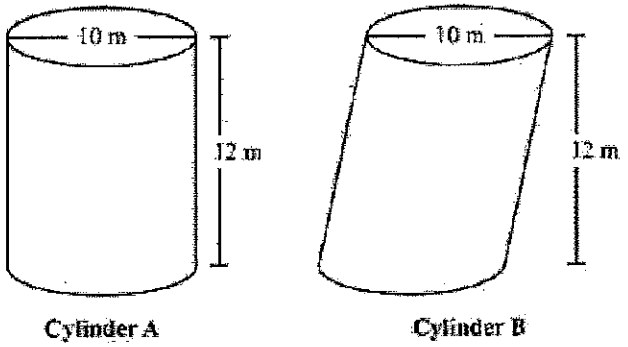


10cm is not the height.., do pyth. Th. first to get height.

$6^2 + b^2 = 10^2$
$36 + b^2 = 100$
$-36 \quad -36$
$b^2 = 64$
$b = \sqrt{64} = 8$

now volume
$V = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi(6)^2(8)$
$= 96\pi\ cm^3$
or $= 301.593\ cm^3$

14. Cylinder A and Cylinder B are shown below. What is the volume of each cylinder? What do you notice about their volume?



Cylinder A Cylinder B

For both . . .

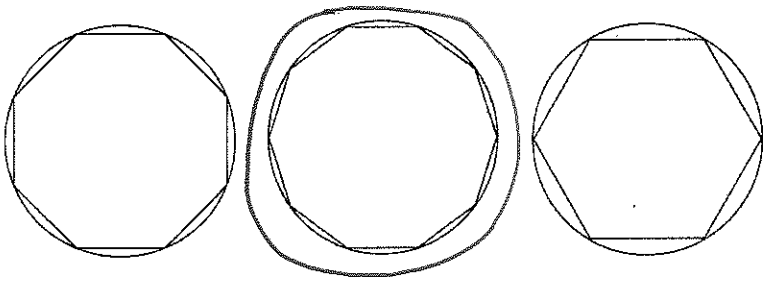
$V = \pi r^2 h$

$= \pi (5)^2 (12)$

$= 300\pi \text{ m}^3$ or 942.478 m^3

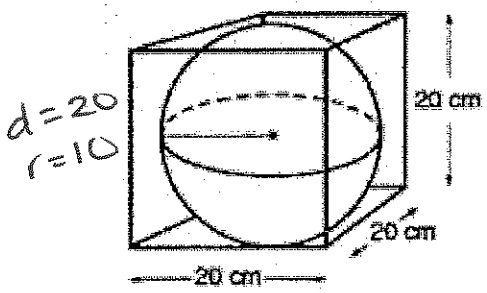
volumes are the same regardless of one being slanted.

15. Which of the following inscribed shapes would have the closest perimeter to pi? Explain.



The 2nd has a perimeter closest to pi because the more sides the inscribed shape has the closer it gets to the circular shape.

16. Using the figure below, determine the volume of the sphere.



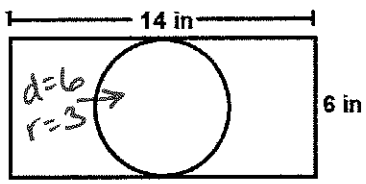
$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi (10)^3$

$= 4188.790 \text{ cm}^3$

ends up with 6 equilateral Δ's.

17. Find the area of the circle.



A of Circle

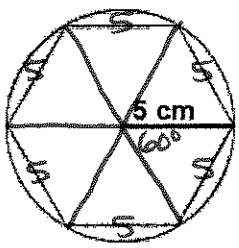
$A = \pi r^2$

$= \pi (3)^2$

$= 9\pi \text{ in}^2$

or $= 28.274 \text{ in}^2$

18. Find the perimeter of the inscribed hexagon.



~~Hint: use trig to find 1 side of the hexagon.~~

6 sides so $\frac{360}{6} = 60°$

$P = 5(6) = 30 \text{ cm}$