

NAME: \_\_\_\_\_ PERIOD: \_\_\_\_\_

# Algebra II

## UNIT 4

### Radical and Rational Functions

## UNIT 4 – Graphing Radical and Rational Functions

### WHAT ARE YOU LEARNING?

#### Henry County Graduate Learner Outcomes:

- As a Henry County graduate, I will be able to create, interpret, use, and analyze patterns of algebraic structures to make sense of problems.
- As a Henry County graduate, I will be able to use functions to interpret and analyze a variety of contexts.

#### Georgia Standards of Excellence:

##### **Lesson 4-1 – Graphing Radical Functions**

MGSE9-12.F.IF.7 Graph functions expressed algebraically and show key features of the graph both by hand and by using technology.

MGSE9-12.F.IF.7b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

##### **Lesson 4-2 – Solving Radical Equations**

MGSE9-12.A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

##### **Lesson 4-3 – Graphing Rational Functions**

MGSE9-12.F.IF.7d Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

MGSE9-12.F.IF.4 Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features.

MGSE9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

##### **Lesson 4-4 – Simplifying, Multiplying, and Dividing Rational Expressions**

##### **Lesson 4-5 – Adding and Subtracting Rational Expressions**

MGSE9-12.A.APR.7 Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

##### **Lesson 4-6 – Solving Rational Equations**

MGSE9-12.A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

MGSE9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from simple rational functions.

## UNIT 4 – Graphing Radical and Rational Functions

**WHY ARE YOU LEARNING THIS?**

### Level 3 Performance Task:

Students will create their own rational equation and identify all characteristics studied in this unit.

**WHAT IS YOUR GOAL FOR THIS UNIT?**

Unit Goal:

I scored a \_\_\_\_\_ on my pretest.

My goal is to score a \_\_\_\_\_ or higher on the end of unit test.

To achieve this goal I will \_\_\_\_\_

**HOW WILL YOU KNOW WHEN YOU'VE MASTERED THIS? SHOW ME THE EVIDENCE!**

Data Analysis: Pre-Test Score \_\_\_\_\_ Post-Test Score \_\_\_\_\_

Learning Targets:	Pre-Test Score	Quiz Score	Post-Test Score
R1: I can graph square root functions and identify characteristics of the graph. (F.IF.7b)			
R2: I can graph cube root functions and identify characteristics of the graph.. (F.IF.7b)			
R3: I can solve simple radical equations. (A.REI.2)			
R4: I can solve simple radical inequalities. (A.CED.1)	____	____	____
R8: I can graph rational functions. (F.IF.7d, F.IF.4, F.IF. 5)			
R9: I can identify characteristics of a rational function. (F.IF.7d, F.IF.4, F.IF.5)			
R5: I can simplify, multiply, and divide rational expressions. (A.ARP.7)			
R6: I can add and subtract rational expressions. (A.ARP.7)			
R7: I can solve simple rational equations. (A.REI.2)			

## UNIT 4 – Graphing Radical and Rational Functions

### LEARNING ACTIVITIES

#### **Lesson 4-1 – Graphing Radical Functions**

R1: I can graph square root functions and identify characteristics of the graph.. (F.IF.7b)

R2: I can graph cube root functions and identify characteristics of the graph.. (F.IF.7b)

- Complete guided notes with teacher    **OR**    Watch video lesson and take notes  
 Complete 4-1 practice

#### **Lesson 4-2 – Solving Radical Equations**

R3: I can solve simple radical equations. (A.REI.2)

R4: I can solve simple radical inequalities. (A.CED.1)

- Complete guided notes with teacher    **OR**    Watch video lesson and take notes  
 Complete 4-2 practice

#### **Lesson 4-3 – Graphing Rational Functions**

R8: I can graph rational functions. (F.IF.7d, F.IF.4, F.IF. 5)

R9: I can identify characteristics of a rational function. (F.IF.7d, F.IF.4, F.IF.5)

- Complete guided notes with teacher    **OR**    Watch video lesson and take notes  
 Complete 4-3 practice

#### **Lesson 4-4 – Simplifying, Multiplying, and Dividing Rational Expressions**

R5: I can simplify, multiply, and divide rational expressions. (A.APR.7)

- Complete guided notes with teacher    **OR**    Watch video lesson and take notes  
 Complete 4-4 practice

#### **Lesson 4-5 – Adding and Subtracting Rational Expressions**

R6: I can add and subtract rational expressions. (A.APR.7)

- Complete guided notes with teacher    **OR**    Watch video lesson and take notes  
 Complete 4-5 practice

#### **Lesson 4-6 – Solving Rational Equations**

R7: I can solve simple rational equations. (A.REI.2)

- Complete guided notes with teacher    **OR**    Watch video lesson and take notes  
 Complete 4-6 practice

### UNIT 4 Assessments

- Complete Unit 4 Performance Task  
 Complete Unit 4 Review Guide  
 Complete Unit 4 Test  
 Complete Unit 4 Reflection

## Lesson 4-1: Graphs of Radical Functions

Learning Target: R1: \_\_\_\_\_

R2: \_\_\_\_\_

Consider the following EQUATIONS, make a table, plot the points, and graph what you think the graph looks like.

1.  $f(x) = \sqrt{x}$

2.  $f(x) = 2\sqrt{x}$

3.  $f(x) = 0.5\sqrt{x}$

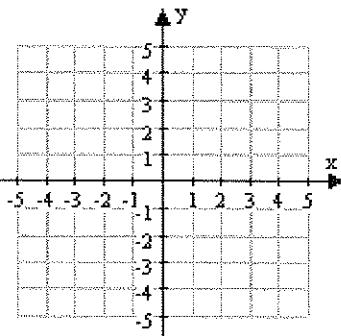
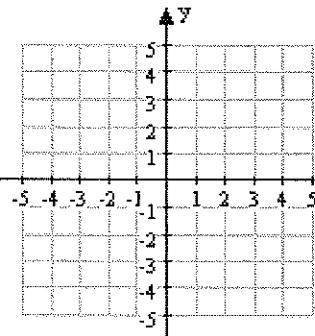
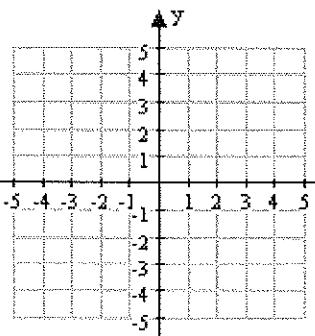
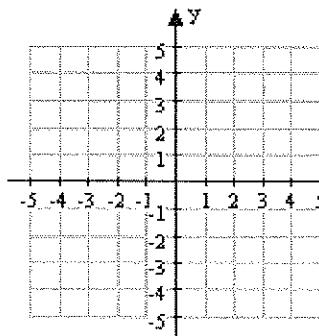
4.  $f(x) = -2\sqrt{x}$

x	y
-4	
-1	
0	
1	
2	
3	
4	

x	y
-4	
-1	
0	
1	
2	
3	
4	

x	y
-4	
-1	
0	
1	
2	
3	
4	

x	y
-4	
-1	
0	
1	
2	
3	
4	



Domain:

Domain:

Domain:

Domain:

Range:

Range:

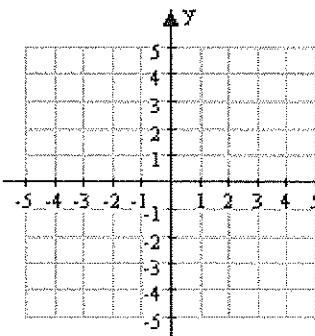
Range:

Range:

5. What happens to the graph as the number in front of  $\sqrt{x}$  gets Larger? Close to Zero? Negative?

6.  $f(x) = \sqrt{-x}$

$x$	$y$
-4	
-3	
-2	
-1	
0	
1	
4	

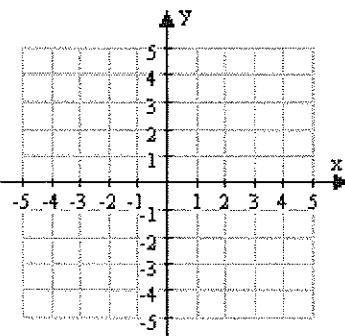


Domain:

Range:

7.  $f(x) = \sqrt{x+4}$

$x$	$y$
-5	
-4	
-3	
-2	
0	
1	
5	

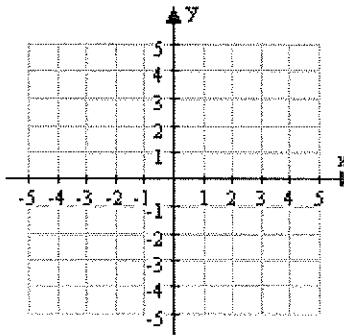


Domain:

Range:

8.  $f(x) = \sqrt{x-1}$

$x$	$y$
-3	
0	
1	
2	
3	
4	
5	

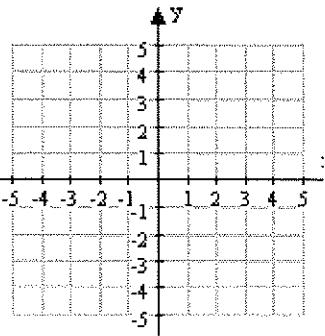


Domain:

Range:

9.  $f(x) = \sqrt{x+4} + 3$

$x$	$y$
-5	
-4	
-3	
-2	
0	
1	
3	
5	

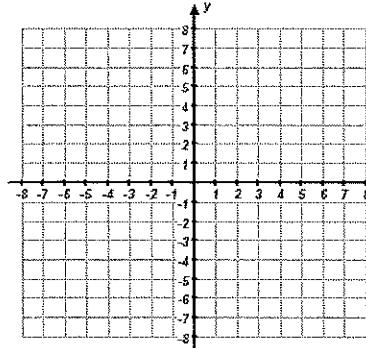


Domain:

Range:

10.  $f(x) = \sqrt[3]{x}$

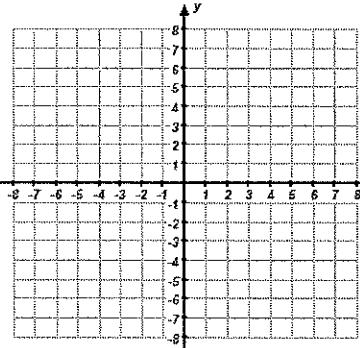
$x$	$y$
-8	
-4	
-1	
0	
1	
4	
8	



Domain:

Range:

$x$	$y$
-8	
-4	
-1	
0	
1	
4	
8	

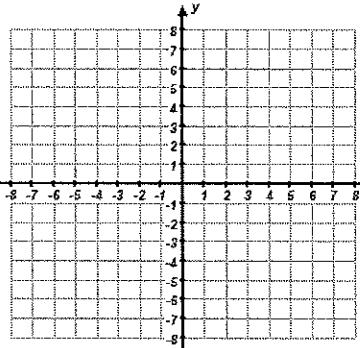


Domain:

Range:

12.  $f(x) = \sqrt[3]{x+3}$

$x$	$y$
-8	
-4	
-3	
-2	
1	
5	
8	

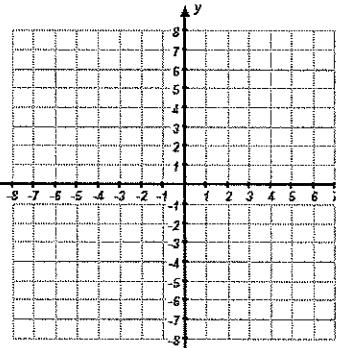


Domain:

Range:

13.  $f(x) = \sqrt[3]{x+3} + 4$

$x$	$y$
-8	
-4	
-3	
-2	
1	
5	
8	



Domain:

Range:

**Take Aways... What did you notice?**

**Positive Square Roots:**

$$f(x) = 4\sqrt{x}$$

D:  $[0, \infty)$   
R:  $[0, \infty)$

$$f(x) = 4\sqrt{x+3}$$

D:  $[-3, \infty)$   
R:  $[0, \infty)$

$$f(x) = 4\sqrt{x} - 5$$

D:  $[0, \infty)$   
R:  $[-5, \infty)$

**Rule:**      **Domain  $\rightarrow$**

**Range  $\rightarrow$**

**Negative Square Roots:**

$$f(x) = -4\sqrt{x}$$

D:  $[0, \infty)$   
R:  $(-\infty, 0]$

$$f(x) = -4\sqrt{x+3}$$

D:  $[-3, \infty)$   
R:  $(-\infty, 0]$

$$f(x) = -4\sqrt{x} - 5$$

D:  $[0, \infty)$   
R:  $(-\infty, -5]$

**Rule:**      **Domain  $\rightarrow$**

**Range  $\rightarrow$**

**Cubed Roots:**

**Rule:**      **Domain  $\rightarrow (-\infty, \infty)$**

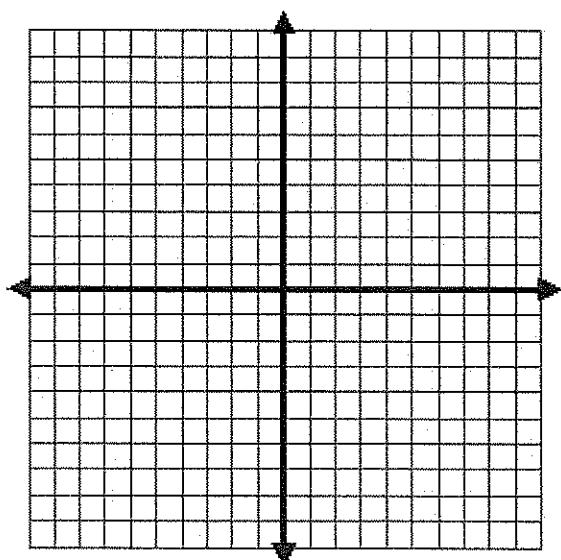
**Range  $\rightarrow (-\infty, \infty)$**

**You Try:**

1.  $f(x) = 4\sqrt{x}$

x	f(x)
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	

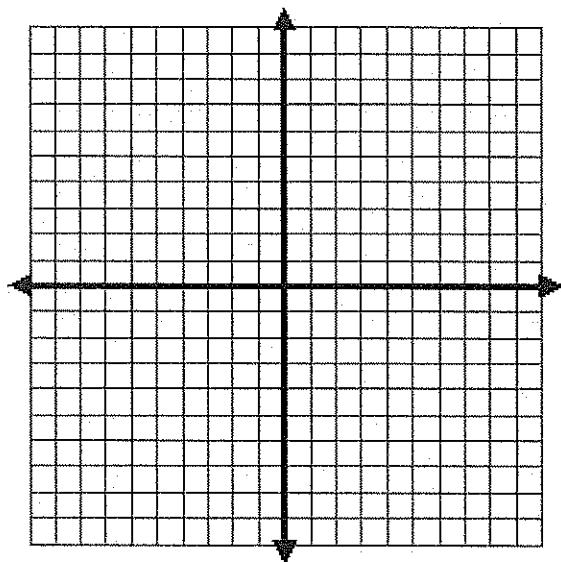
Domain:



2.  $f(x) = 4\sqrt{x + 3}$

x	f(x)
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	

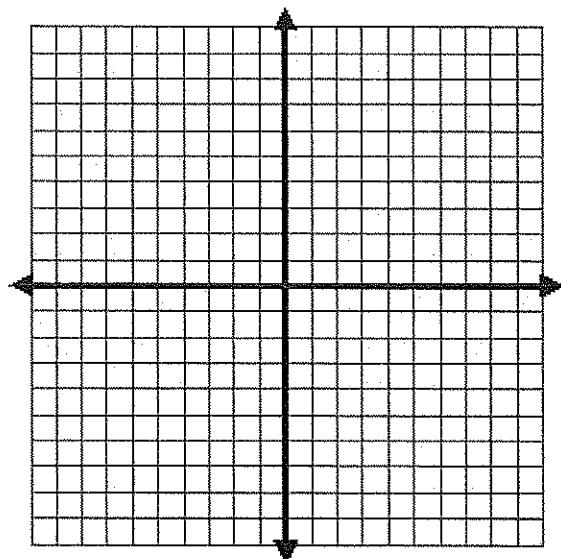
Domain:



3.  $f(x) = 4\sqrt{x - 5}$

x	f(x)
-1	
0	
1	
2	
3	
4	
5	
6	

Domain:

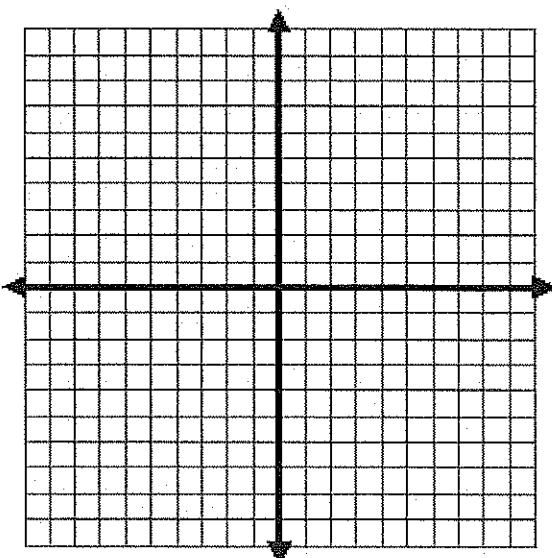


$$4. \ f(x) = -2\sqrt{x+3} + 5$$

x	f(x)
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	

Domain:

Range:

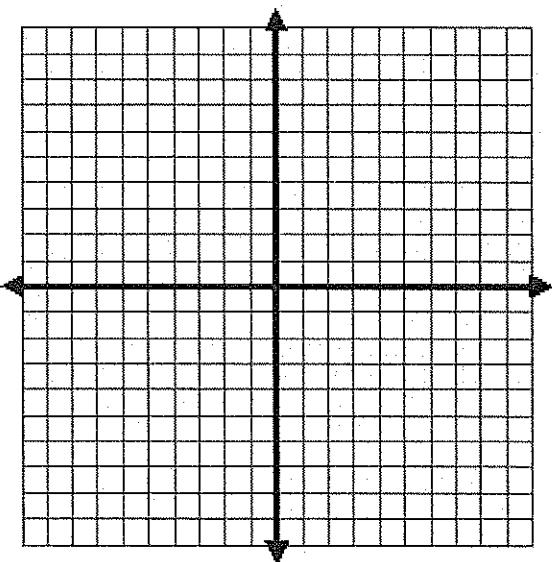


$$5. \ f(x) = \sqrt[3]{x-1} + 4$$

x	f(x)
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	

Domain:

Range:

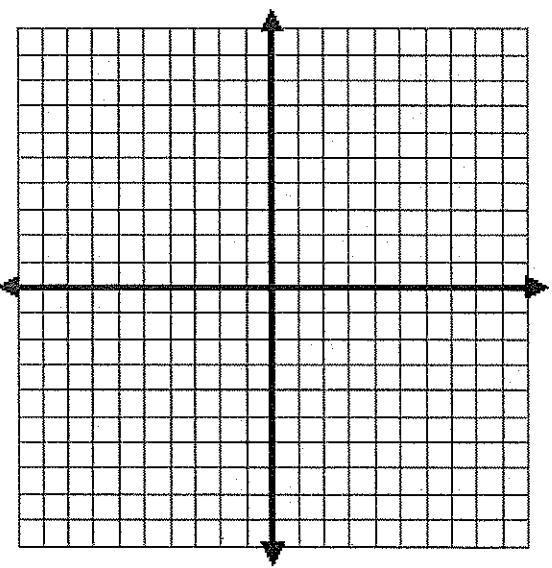


$$6. \ f(x) = 3\sqrt[3]{x}.$$

x	f(x)
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	

Domain:

Range:

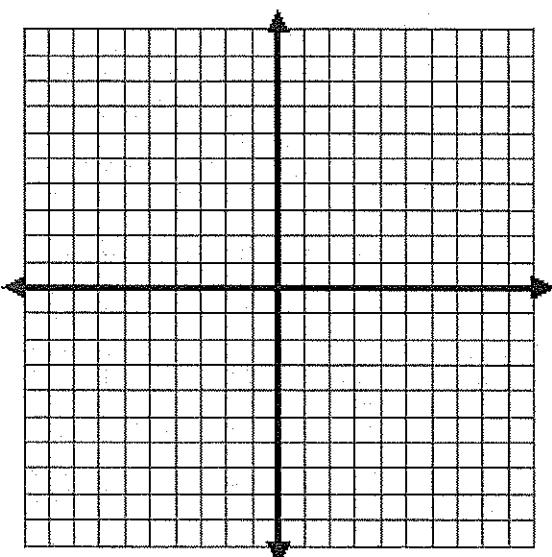


7.  $f(x) = 3\sqrt[3]{x} + 2$

x	f(x)
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	

Domain:

Range:

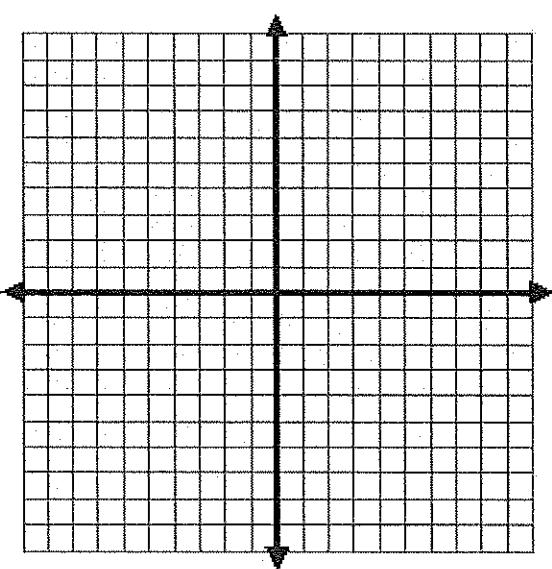


8.  $f(x) = 3\sqrt[3]{x - 2}$

x	f(x)
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	

Domain:

Range:

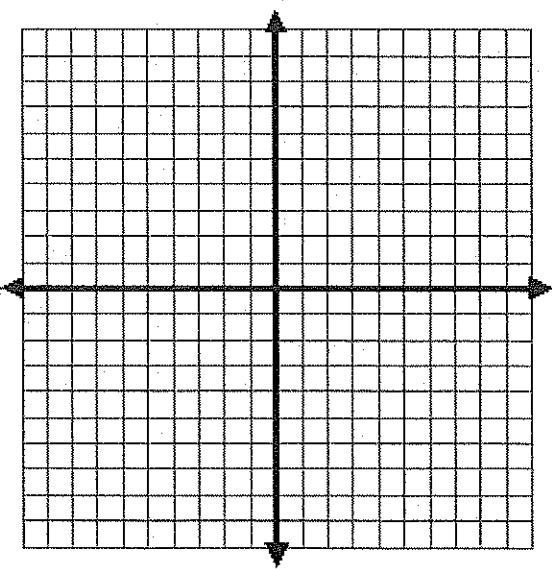


9.  $f(x) = -\sqrt[3]{x - 1} + 4$

x	f(x)
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	

Domain:

Range:



## Lesson 4-2: Solving Radical Equations and Inequalities

Learning Target: R3: \_\_\_\_\_

R4: \_\_\_\_\_

Find the value for the variable that makes the statement true. (SHOW WORK NEATLY)

1.  $8\sqrt{2x+3} - 10 = 30$

2.  $2\sqrt{x+4} + 9 = 1$

3.  $3\sqrt{4x} - 5 = 13$

4.  $12 = 2\sqrt{2(x+1)} - 4$

5.  $\sqrt[3]{2x+1} + 2 = 5$

6.  $\sqrt[3]{2x} - 10 = -6$

**One Radical Basic**

I. Isolate the Radical if possible.

*Example*

$$\begin{aligned} 3\sqrt{x+2} + 4 &= 19 \\ -4 &\quad -4 \\ \hline 3\sqrt{x+2} &= 15 \\ \cancel{3} &\quad \cancel{3} \\ \sqrt{x+2} &= 5 \end{aligned}$$

II. Square Both Sides

*Example*

$$(\sqrt{x+2})^2 = (5)^2$$

$$x+2 = 25$$

III. Isolate the Variable

*Example*

$$\begin{aligned} x+2 &= 25 \\ -2 &\quad -2 \\ \hline x &= 23 \end{aligned}$$

IV. Must Verify the Solution  
(This is not optional some solutions are extraneous.)

*Example*

$$\begin{aligned} 3\sqrt{(23)+2} + 4 &= 19 \\ 3\sqrt{25} + 4 &= 19 \\ 3(5) + 4 &= 19 \\ 15 + 4 &= 19 \\ 19 &= 19 \checkmark \end{aligned}$$

Find the value for the variable that makes the statement true. (SHOW WORK NEATLY)

7.  $\sqrt{5x+2} = \sqrt{3x+12}$

8.  $2\sqrt{x-5} = \sqrt{3x+2}$

9.  $2\sqrt{5x-4} = 3\sqrt{x+8}$

10.  $2\sqrt[3]{5x-3} = \sqrt[3]{35x+6}$

11.  $x+3 = \sqrt{15+x}$

Find the value for the variable that makes the statement true. (SHOW WORK NEATLY)

12.  $x-1 = \sqrt{5x-9}$

13.  $2x+1 = \sqrt{11-2x}$

### Two Radical Basic Equation

#### I. Square Both Sides

$$\sqrt{5x+2} = 3\sqrt{x-2}$$

$$(\sqrt{5x+2})^2 = (3\sqrt{x-2})^2$$

$$5x+2 = 9(x-2)$$

#### II. Eliminate Parenthesis

$$5x+2 = 9(x-2)$$

$$5x+2 = 9x-18$$

#### III. Move variables to one side and constants to the other

$$\begin{array}{rcl} 5x+2 & = & 9x-18 \\ -5x & & -5x \\ \hline 2 & = & 4x-18 \\ +18 & & +18 \\ \hline 20 & = & 4x \end{array}$$

#### IV. Divide both sides by the coefficient

$$\begin{array}{rcl} 20 & = & 4x \\ \frac{20}{4} & = & \frac{4x}{4} \\ 5 & = & x \end{array}$$

#### V. Must Verify the Solution

(This is not optional some solutions are extraneous.)

$$\begin{array}{l} \sqrt{5(5)+2} = 3\sqrt{(5)-2} \\ \sqrt{25+2} = 3\sqrt{3} \\ \sqrt{27} = 3\sqrt{3} \\ 3\sqrt{3} = 3\sqrt{3} \checkmark \end{array}$$

**You Try: Be sure to check each solution.**

$$1. \sqrt{4+3x} = 10$$

$$2. \sqrt{2x+1} = 7$$

$$3. \sqrt[3]{4x-1} = 3$$

$$4. \sqrt[3]{8x+3} - 5 = -2$$

$$5. \sqrt[3]{2x+4} = 2\sqrt[3]{3-x}$$

$$6. \sqrt{7x-8} = \sqrt{5x}$$

$$7. \sqrt{3x+5} = \sqrt{x+15}$$

$$8. \sqrt[3]{2x+16} = \sqrt[3]{6x+8}$$

$$9. \sqrt{3x+1} + 7 = 3$$

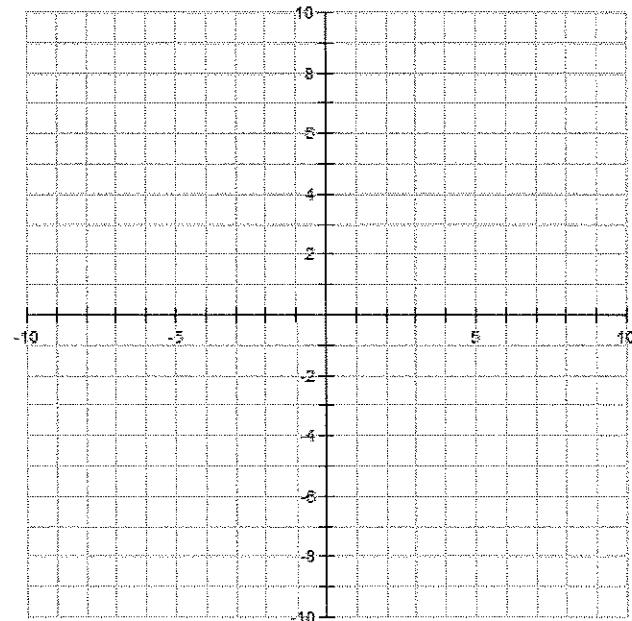
Graph the following and identify the domain and range for each:

1.  $f(x) = \sqrt{x - 3} - 2$

x	y
1	
2	
3	
4	
7	
10	

Domain: \_\_\_\_\_

Range: \_\_\_\_\_

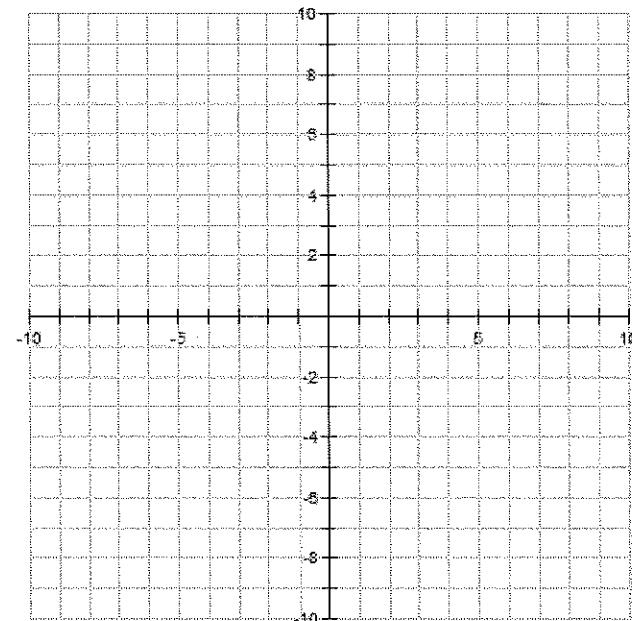


2.  $f(x) = -3\sqrt{x + 1}$

x	y
-3	
-2	
-1	
0	
3	
8	

Domain: \_\_\_\_\_

Range: \_\_\_\_\_

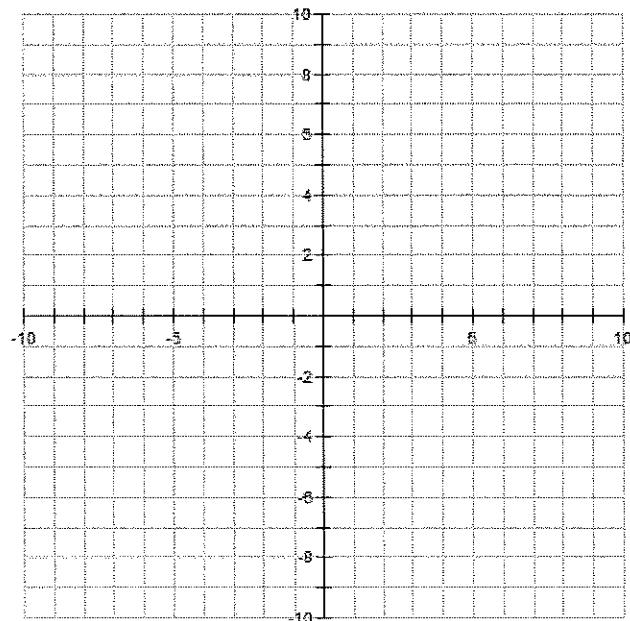


3.  $f(x) = \sqrt[3]{x+2} - 4$

x	y
-10	
-6	
-3	
-2	
-1	
3	
6	
10	

Domain: \_\_\_\_\_

Range: \_\_\_\_\_

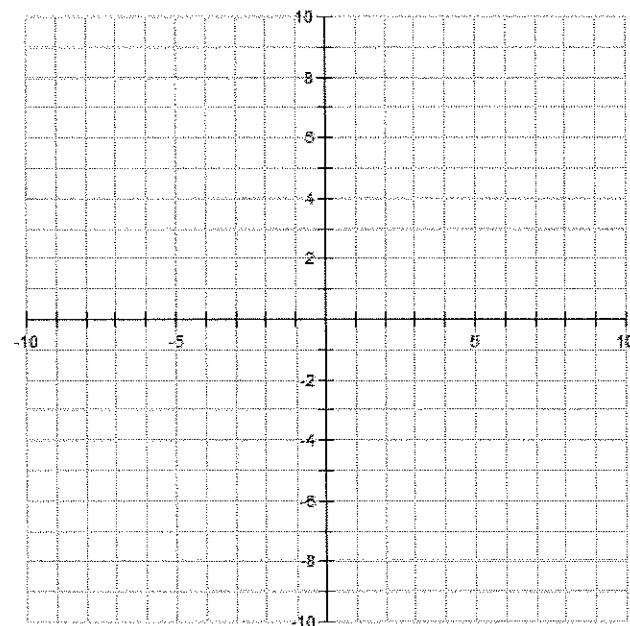


4.  $f(x) = \sqrt[3]{x-2}$

x	y
-10	
-6	
-2	
1	
2	
3	
6	
10	

Domain: \_\_\_\_\_

Range: \_\_\_\_\_



**Identify the domain and range algebraically:**

5.  $f(x) = \sqrt{x-18} + 4$

Domain: \_\_\_\_\_

Range: \_\_\_\_\_

6.  $f(x) = \sqrt[3]{x-8} + 7$

Domain: \_\_\_\_\_

Range: \_\_\_\_\_

7.  $f(x) = \sqrt{x+1} - 7$

Domain: \_\_\_\_\_

Range: \_\_\_\_\_

8.  $f(x) = \sqrt[3]{x+3} - 11$

Domain: \_\_\_\_\_

Range: \_\_\_\_\_

**Solve each of the following equations. Show organized work to support your answers.  
Be sure to check for extraneous solutions.**

$$17. \sqrt{v+4} - 3 = 8$$

$$18. \sqrt{4n} = \sqrt{6x-4}$$

$$19. \sqrt{2x+15} = \sqrt{5x-9}$$

$$20. \sqrt{p+8} = 1$$

$$21. \sqrt[4]{3x+2} = \sqrt[4]{2x+7}$$

$$22. \sqrt[3]{2x+1} = \sqrt[3]{10}$$

$$23. 2\sqrt{x+5} + 11 = 1$$

$$24. \sqrt[3]{3x+5} + 3 = 6$$

$$25. 4\sqrt{2x+3} - 6 = 22$$

$$26. 2\sqrt[3]{1-4x} + 5 = 7$$

## Lesson 4-3: Graphing Rational Expressions

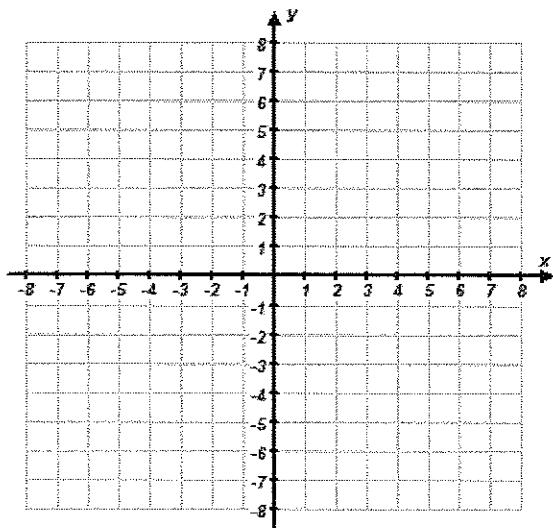
Learning Target: R8: \_\_\_\_\_

R9: \_\_\_\_\_

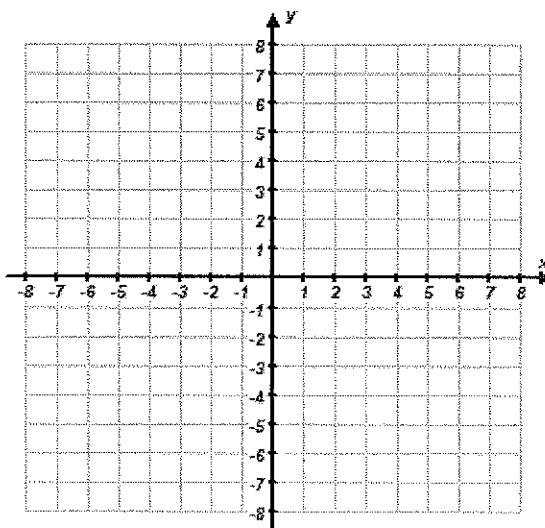
Characteristic	Description	Example
<b>Hole</b> <i>(Point Discontinuity)</i>	A hole <b>usually</b> occurs in the graph of a rational function when a linear factor in the numerator and denominator "divide out". The result is the same as the graph of the simplified function but with a missing point in the graph.	<p>Graph: <math>f(x) = \frac{x^2 - 3x + 2}{x - 2}</math></p> <p>Simplifies: <math>f(x) = \frac{(x-2)(x-1)}{(x-2)}</math></p> <p>This will create a hole at <math>x = 2</math>.</p>
<b>Vertical Asymptote</b> <i>(Infinite Discontinuity)</i>	A vertical asymptote occurs any time a linear factor of the denominator doesn't "divide out" with a factor in the numerator.	<p>Graph: <math>f(x) = \frac{x+1}{x^2+x-2}</math></p> <p>Simplifies: <math>f(x) = \frac{(x+1)}{(x+2)(x-1)}</math></p> <p>This will create a vertical asymptote at <math>x = -2</math>.</p> <p>This will create a vertical asymptote at <math>x = 1</math>.</p>
<b>Vertical Asymptote &amp; Hole</b>	To have both a hole and a vertical asymptote the rational function must have at least one linear factor that divides out and one linear factor that does not.	<p>Graph: <math>f(x) = \frac{x^2+x-2}{x^2-1}</math></p> <p>Simplifies: <math>f(x) = \frac{(x+2)(x-1)}{(x+1)(x-1)}</math></p> <p>This will create a vertical asymptote at <math>x = -1</math>.</p> <p>This will create a hole at <math>x = 1</math>.</p>

Sketch a graph of the following rational functions. Label any holes or vertical asymptotes. Use your calculator for additional assistance.

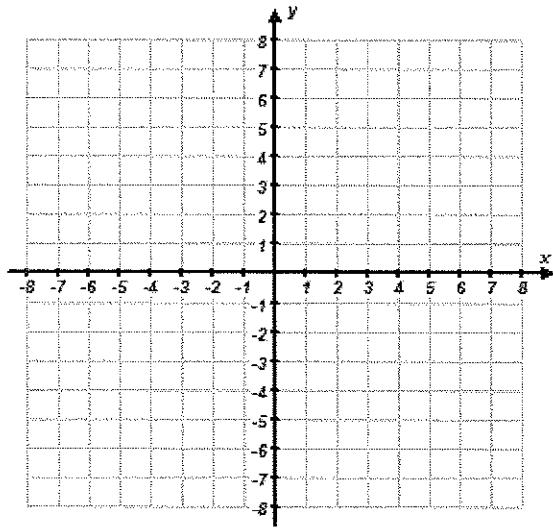
1.  $f(x) = \frac{x^2+2x-3}{x-1}$



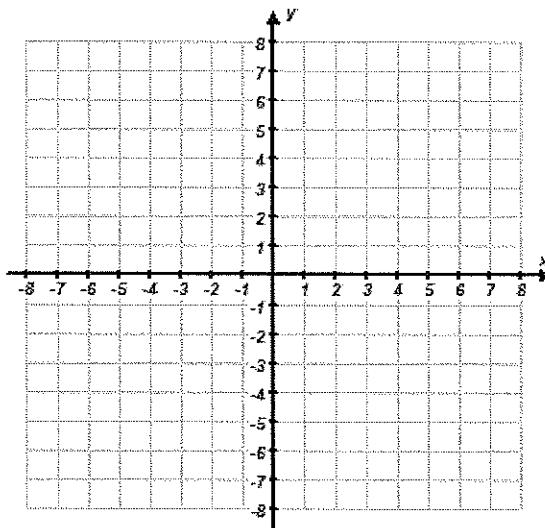
2.  $f(x) = \frac{x^2+3x-4}{x^2+2x-8}$



3.  $f(x) = \frac{x^2+x-6}{x+3}$



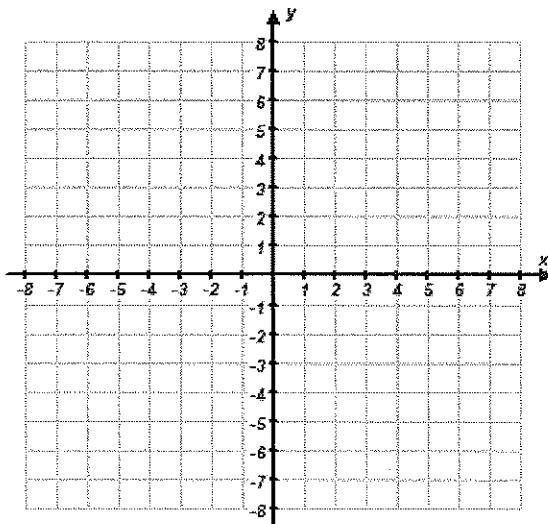
4.  $f(x) = \frac{x^2-4}{x^2+6x+8}$



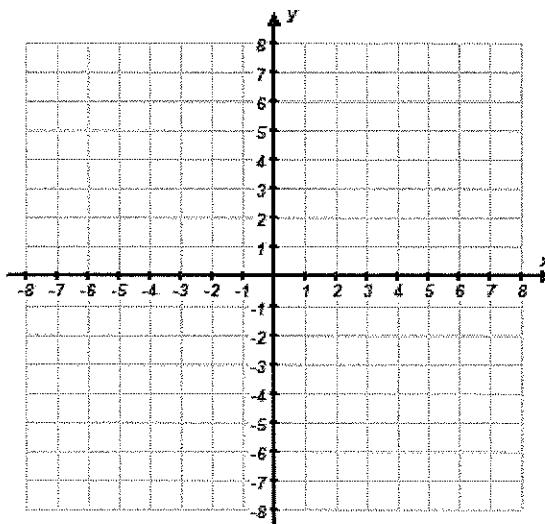
Potential Horizontal Asymptotes	Description	Example
<b>Case #1:</b>	A rational function that has a numerator polynomial with the <b>same degree</b> as the polynomial in the denominator creates a horizontal asymptote that passes through the y-axis at the quotient of the leading coefficients.	<p>Graph: <math>f(x) = \frac{6x^3 + x^2 - 2}{2x^3 - 2}</math></p> <p>Analyze: <math>f(x) = \frac{\textcircled{1}6x^3 + \textcircled{2}x^2 - 2}{\textcircled{2}2x^3 - 2}</math></p> <p>Asymptote: <math>y = \frac{6}{2}</math> or <math>y = 3</math></p>
<b>Case #2:</b>	A rational function that has a polynomial in the <b>numerator that has a smaller degree</b> than the degree of the polynomial in the denominator creates a horizontal asymptote at $y = 0$ .	<p>Graph: <math>f(x) = \frac{2x^2 + 1}{3x^3 - 3}</math></p> <p>Asymptote: <math>y = 0</math></p>
<b>Case #3:</b>	A rational function that has a polynomial in the <b>numerator that has a larger degree</b> than the degree of the polynomial in the denominator does not have a horizontal asymptote.	<p>Graph: <math>f(x) = \frac{5x^2 + 3x - 4}{4x^1 + 1}</math></p> <p>Asymptote: No Horizontal Asymptote</p>

Sketch a graph of the following rational functions. Label any vertical asymptotes, horizontal asymptotes, or holes. Use your calculator for additional assistance.

5.  $f(x) = \frac{x-4}{x^2-2x-8}$

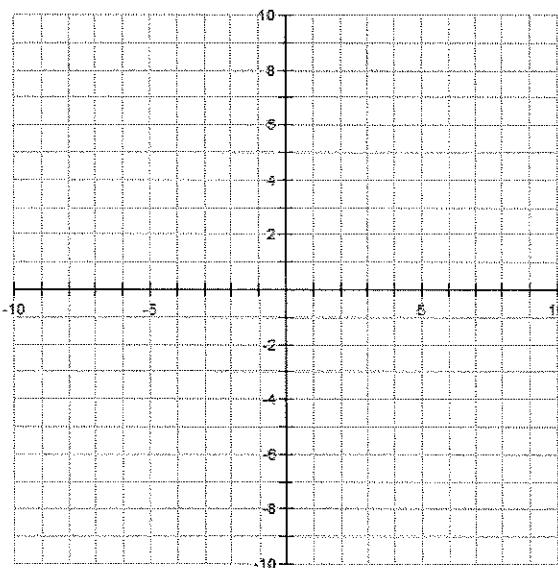


6.  $f(x) = \frac{2x^2-8}{x^2+x-6}$



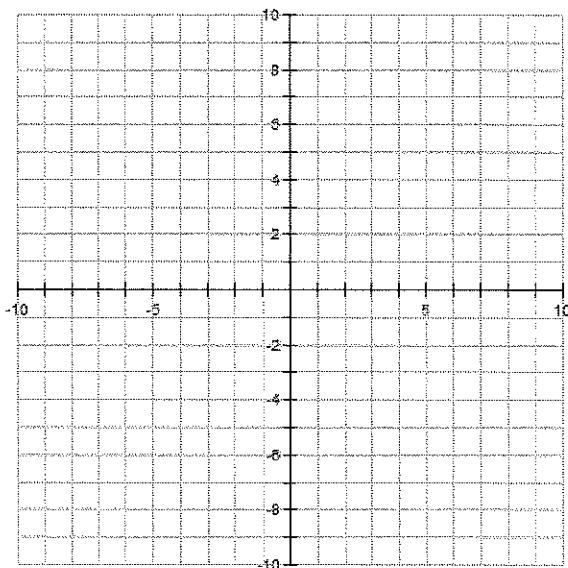
Let's put it all together....

1.)  $f(x) = \frac{x^2-4}{x-2}$



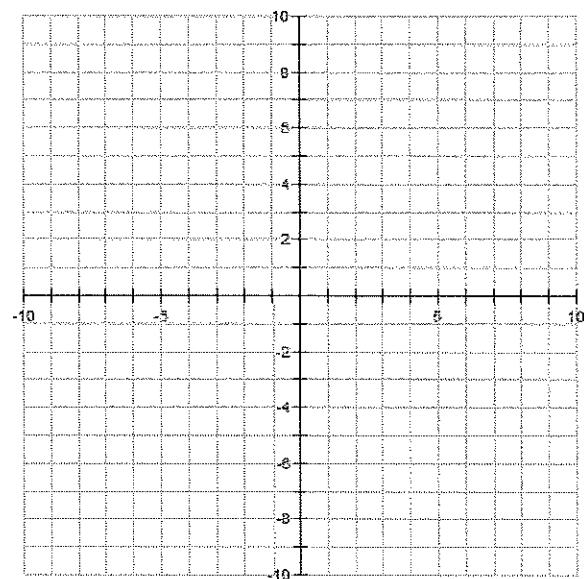
Hole(s)	Vertical Asymptote(s)	Horizontal Asymptote	$x$ -intercept(s)	$y$ -intercept	Domain	Range

2.)  $f(x) = \frac{x^3 + 3x^2 - 10x}{x^2 + 5x} = \frac{x(x+5)(x-2)}{x(x+5)}$



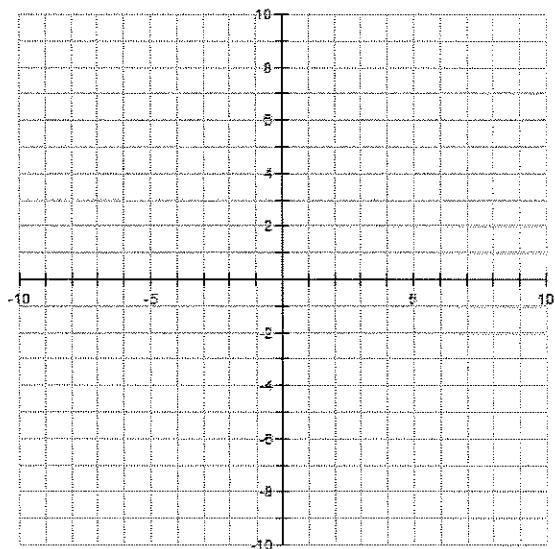
Hole(s)	Vertical Asymptote(s)	Horizontal Asymptote	$x$ -intercept(s)	$y$ -intercept	Domain	Range

3.)  $f(x) = \frac{-5}{x^2 - 2x - 3}$



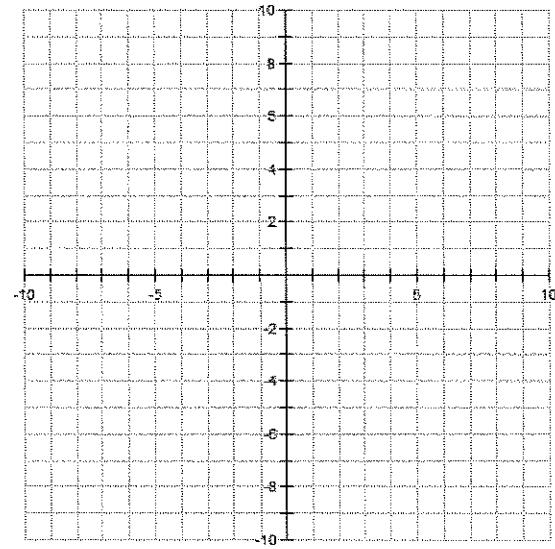
Hole(s)	Vertical Asymptote(s)	Horizontal Asymptote	$x$ -intercept(s)	$y$ -intercept	Domain	Range

4.)  $f(x) = \frac{x^3+4x^2-21x}{x^2+4x-21}$



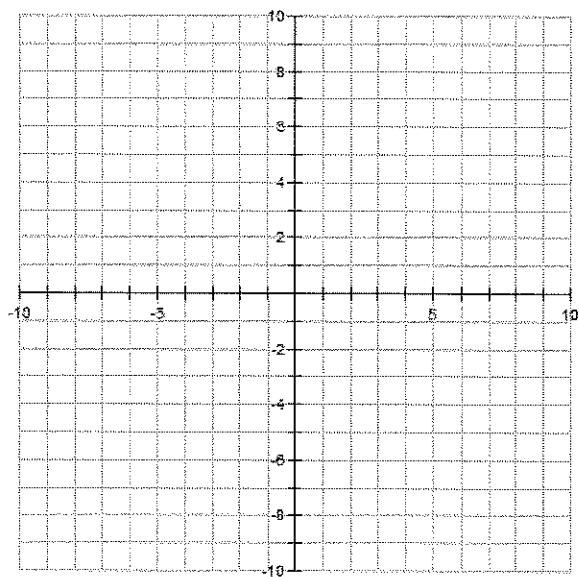
Hole(s)	Vertical Asymptote(s)	Horizontal Asymptote	$x$ -intercept(s)	$y$ -intercept	Domain	Range

5.)  $f(x) = \frac{(x-5)(x-9)(x+1)}{(x-9)(x+1)}$



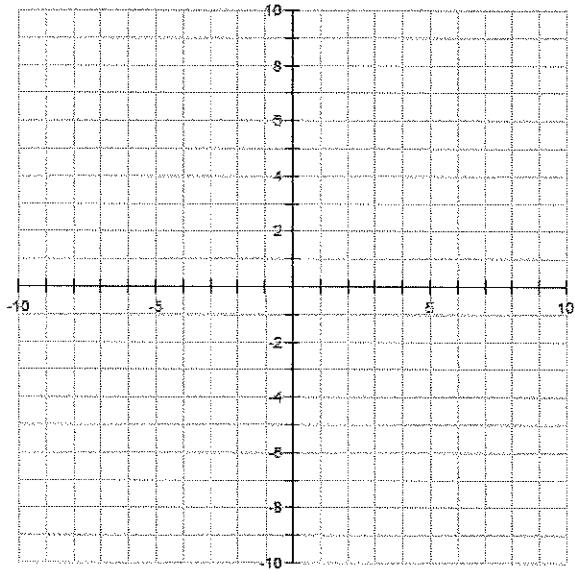
Hole(s)	Vertical Asymptote(s)	Horizontal Asymptote	$x$ -intercept(s)	$y$ -intercept	Domain	Range

6.)  $f(x) = \frac{x^2+x-2}{(x+2)(x^2-2x-15)}$



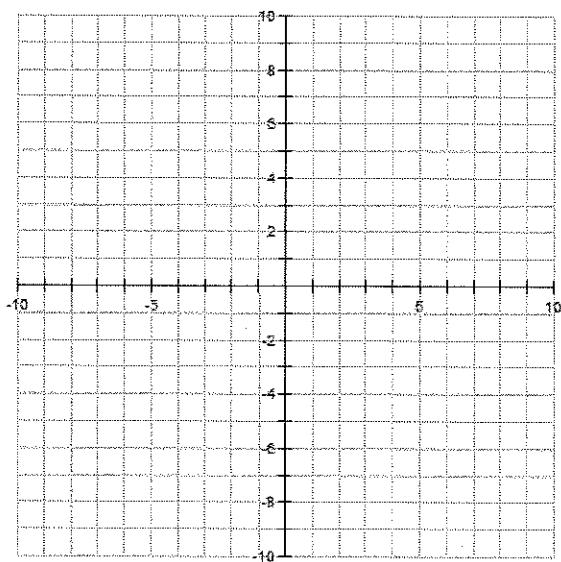
Hole(s)	Vertical Asymptote(s)	Horizontal Asymptote	$x$ -intercept(s)	$y$ -intercept	Domain	Range

7.)  $f(x) = \frac{x^2+x-6}{(x-2)(x^2+5x-36)} = \frac{(x+3)(x-2)}{(x-2)(x+9)(x-4)}$



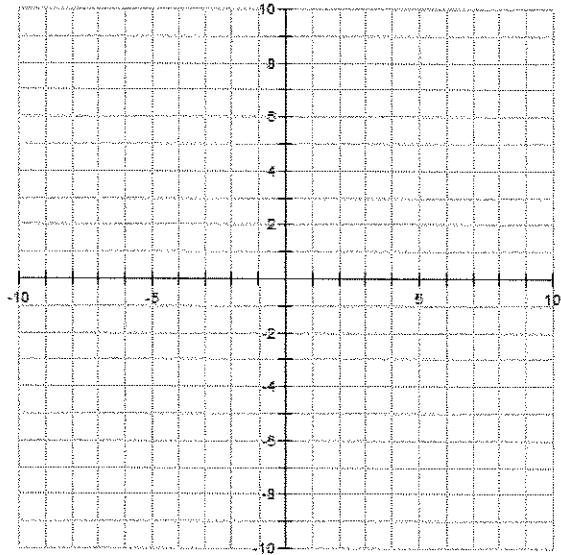
Hole(s)	Vertical Asymptote(s)	Horizontal Asymptote	$x$ -intercept(s)	$y$ -intercept	Domain	Range

8.)  $f(x) = \frac{3x^2}{x^2 - 1}$



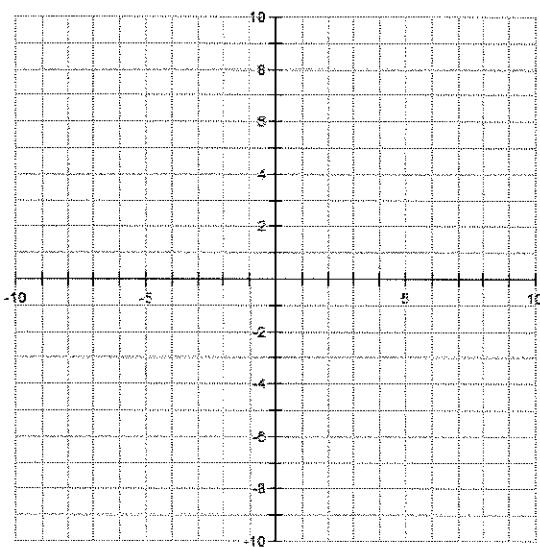
Hole(s)	Vertical Asymptote(s)	Horizontal Asymptote	$x$ -intercept(s)	$y$ -intercept	Domain	Range

9.)  $f(x) = \frac{x^2 + 7x + 12}{x^2 + 11x + 28}$



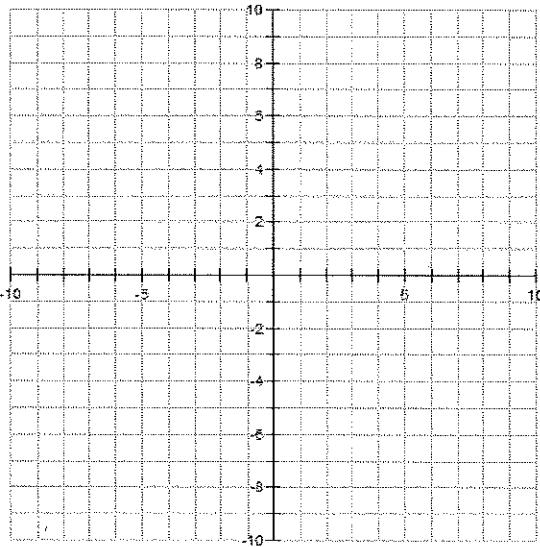
Hole(s)	Vertical Asymptote(s)	Horizontal Asymptote	$x$ -intercept(s)	$y$ -intercept	Domain	Range

10.)  $f(x) = \frac{x^2 - 6x - 7}{x^2 + 3x - 4} = \frac{(x-7)(x+1)}{(x+4)(x-1)}$



Hole(s)	Vertical Asymptote(s)	Horizontal Asymptote	$x$ -intercept(s)	$y$ -intercept	Domain	Range

11.)  $f(x) = \frac{x+8}{x^2+5x-24}$



Hole(s)	Vertical Asymptote(s)	Horizontal Asymptote	$x$ -intercept(s)	$y$ -intercept	Domain	Range

## Lesson 4-3: Graphing Absolute, Step, and Partial Functions

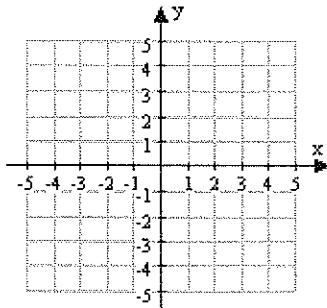
Learning Target: R10: \_\_\_\_\_

R11: \_\_\_\_\_

R12: \_\_\_\_\_

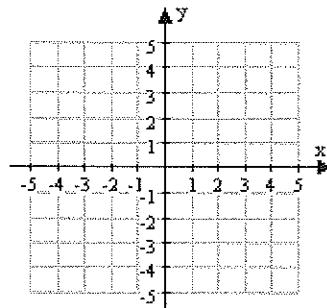
1.  $f(x) = |x|$

$x$	$y$
-4	
-2	
-1	
0	
1	
2	
4	



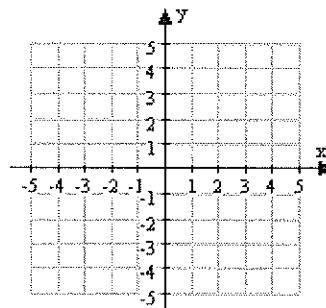
2.  $f(x) = 2|x|$

$x$	$y$
-3	
-2	
-1	
0	
1	
2	
3	



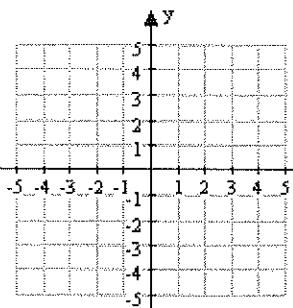
3.  $f(x) = 0.5|x|$

$x$	$y$
-4	
-2	
-1	
0	
1	
2	
3	
4	



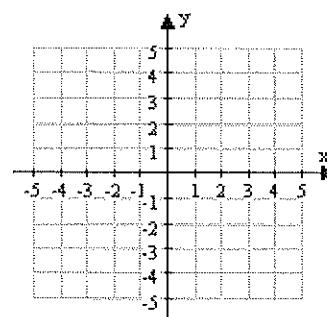
4.  $f(x) = -2|x|$

$x$	$y$
-4	
-1	
0	
1	
2	
3	
4	



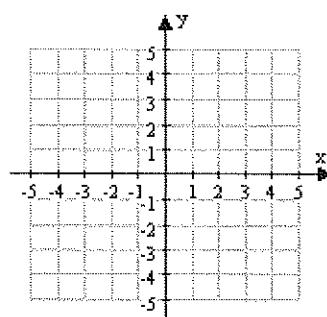
6.  $f(x) = |x| - 3$

$x$	$y$
-4	
-2	
-1	
0	
1	
2	
4	



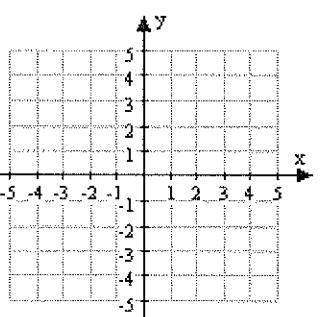
7.  $f(x) = |x+2|$

$x$	$y$
-5	
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	



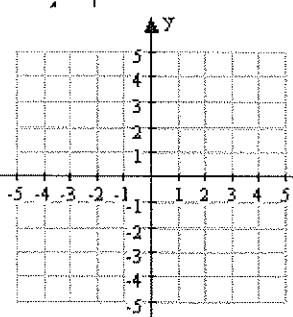
8.  $f(x) = 2|x-1|$

$x$	$y$
-2	
-1	
0	
1	
2	
3	
4	



9.  $f(x) = -|x-2| + 3$

$x$	$y$
-2	
-1	
0	
1	
2	
3	
4	



11.  $f(x) = \lceil x \rceil$

$x$	$y$
-0.5	0
0	0
0.5	0
1	1
1.2	1
2	2
2.4	2

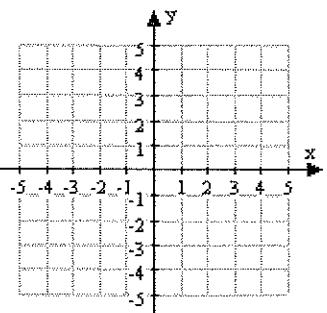
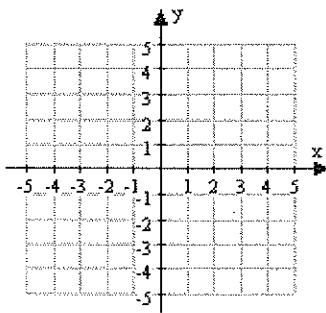
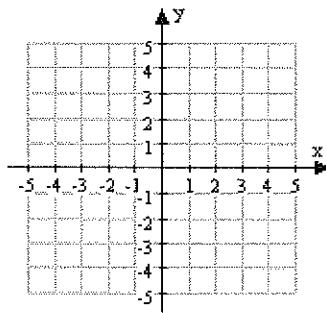
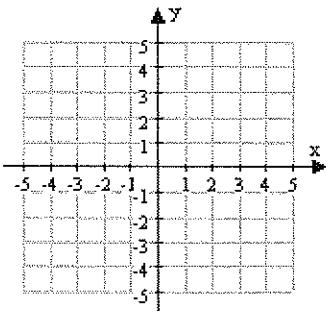
12.  $f(x) = |x|$  or  $f(x) = \llbracket x \rrbracket$

$x$	$y$
-0.5	0.5
0	0
0.5	0.5
1	1
1.2	1.2
2	2
2.4	2.4

$x$	$y$
-0.5	0.5
0	0
0.5	0.5
1	1
1.2	1.2
2	2
2.4	2.4

14.  $f(x) = 2\llbracket x - 1 \rrbracket$

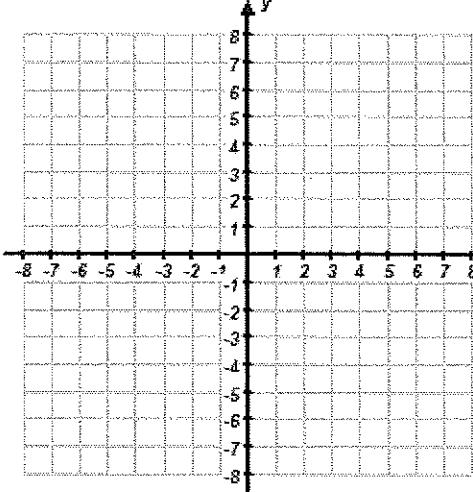
$x$	$y$
-0.5	0
0	0
0.5	0.5
1	1
1.2	1.2
2	2
2.4	2.4



15. Graph the following partial functions (piece-wise).

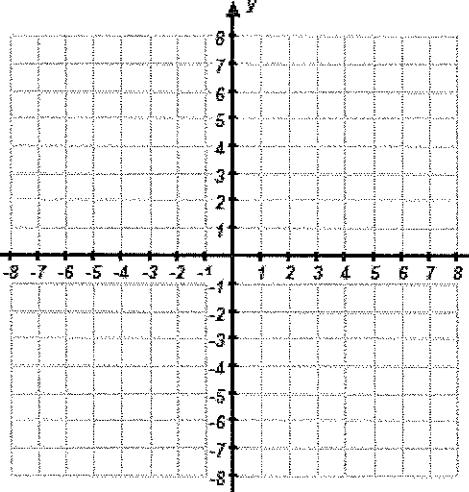
a.

$$f(x) = \begin{cases} -x + 5 & \text{if } x \leq 2 \\ 2x - 1 & \text{if } x > 2 \end{cases}$$



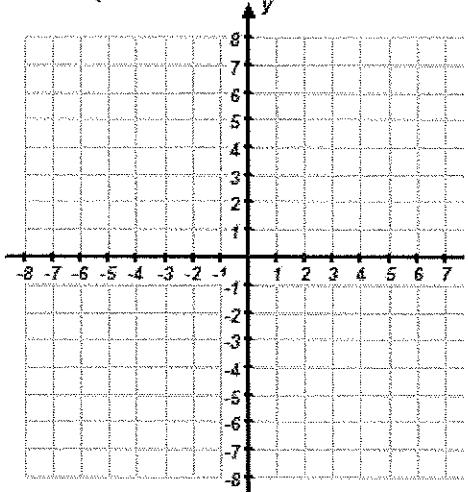
b.

$$f(x) = \begin{cases} x^2 - 3 & \text{if } x \leq 1 \\ -2x + 3 & \text{if } x > 1 \end{cases}$$



c.

$$f(x) = \begin{cases} 3 & \text{if } x < -2 \\ -2x - 1 & \text{if } -2 \leq x \leq 2 \\ x + 1 & \text{if } x > 2 \end{cases}$$



Name: \_\_\_\_\_

Date: \_\_\_\_\_

**Find the vertical & horizontal asymptotes, x & y ints, and holes:**

1.  $f(x) = \frac{1}{x-2}$

hole: \_\_\_\_\_

V.A: \_\_\_\_\_

H.A: \_\_\_\_\_

x-Int.: \_\_\_\_\_

y-int: \_\_\_\_\_

2.  $f(x) = \frac{x^2 - x - 12}{x}$

hole: \_\_\_\_\_

V.A: \_\_\_\_\_

H.A: \_\_\_\_\_

x-Int.: \_\_\_\_\_

y-int: \_\_\_\_\_

3.  $f(x) = \frac{x^2 - 5x + 6}{x^2 - 4x + 3}$

hole: \_\_\_\_\_

V.A: \_\_\_\_\_

H.A: \_\_\_\_\_

x-Int.: \_\_\_\_\_

y-int: \_\_\_\_\_

4.  $f(x) = \frac{x^2 + x}{x + 1}$

hole: \_\_\_\_\_

V.A: \_\_\_\_\_

H.A: \_\_\_\_\_

x-Int.: \_\_\_\_\_

y-int: \_\_\_\_\_

5.  $f(x) = \frac{2x^2 - 4x}{x^2 - 2x - 3}$

hole: \_\_\_\_\_

V.A: \_\_\_\_\_

H.A: \_\_\_\_\_

x-Int.: \_\_\_\_\_

y-int: \_\_\_\_\_

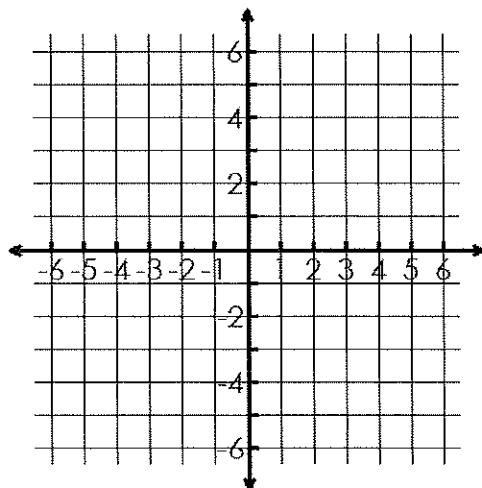
6.  $f(x) = \frac{x+4}{x^2 + 3x - 4}$

Hole: \_\_\_\_\_

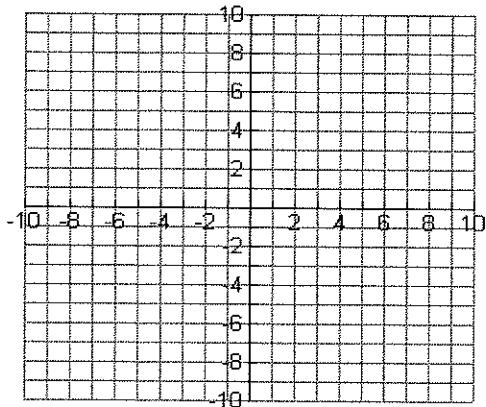
V.A: \_\_\_\_\_ H.A: \_\_\_\_\_

x-Int.: \_\_\_\_\_ y-int: \_\_\_\_\_

D: \_\_\_\_\_ R: \_\_\_\_\_



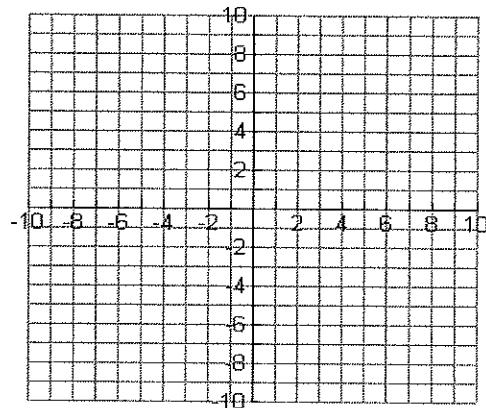
1. Graph  $f(x) = \frac{5}{x+3}$



x-int \_\_\_\_\_ HA\_\_\_\_\_

y-int \_\_\_\_\_ VA\_\_\_\_\_

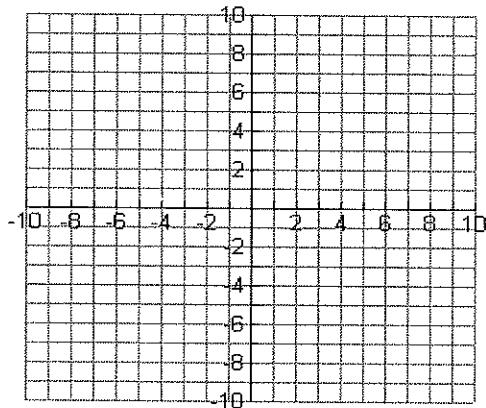
2. Graph  $f(x) = \frac{2x-1}{x+3}$



x-int \_\_\_\_\_ HA\_\_\_\_\_

y-int \_\_\_\_\_ VA\_\_\_\_\_

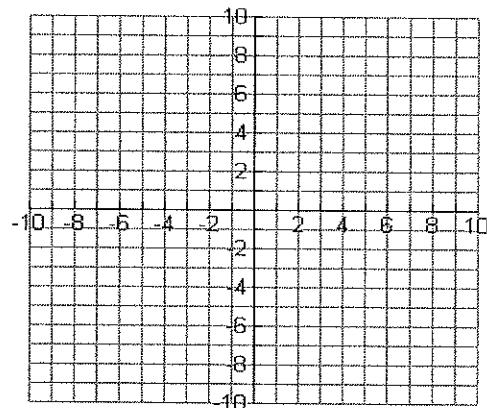
3. Graph  $f(x) = \frac{2x^2 + 3x - 2}{x^2 - 9}$



x-int \_\_\_\_\_ HA\_\_\_\_\_

y-int \_\_\_\_\_ VA\_\_\_\_\_

4. Graph  $f(x) = \frac{3-4x}{x-1}$



x-int \_\_\_\_\_ HA\_\_\_\_\_

y-int \_\_\_\_\_ VA\_\_\_\_\_

**Unit 4 – QUIZ REVIEW – Graphing Rationals**

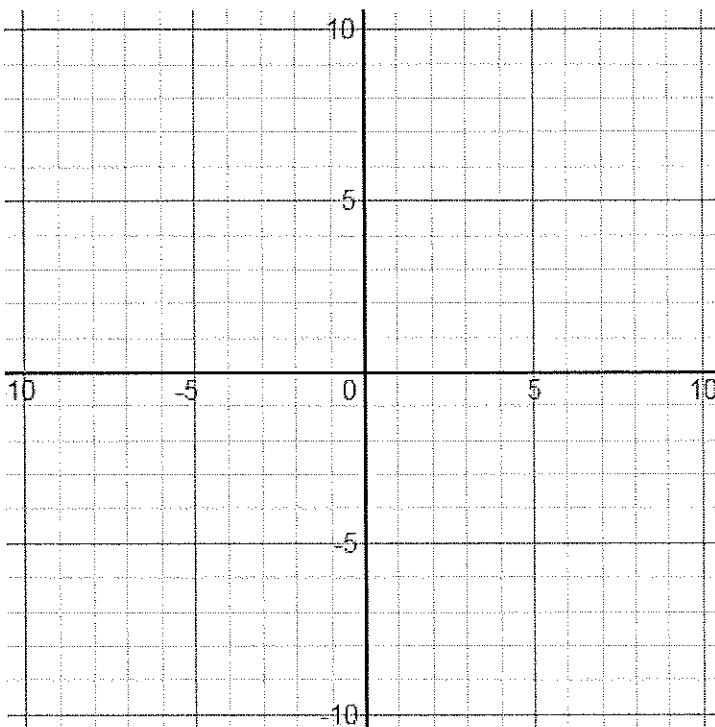
Name: \_\_\_\_\_

R8: I can graph rational functions.

R9: I can identify characteristics of a rational function.

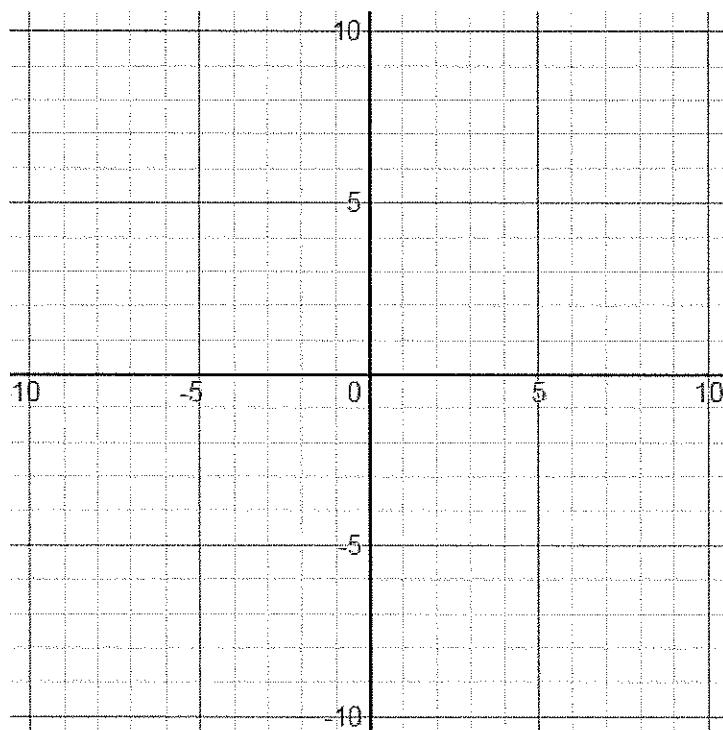
**Graph the following and identify the characteristics:**

1) The graph of  $h(x) = \frac{x^2 - 3x - 4}{x^3 - 4x^2 - 9x + 36} = \frac{(x-4)(x+1)}{(x-4)(x+3)(x-3)}$  =



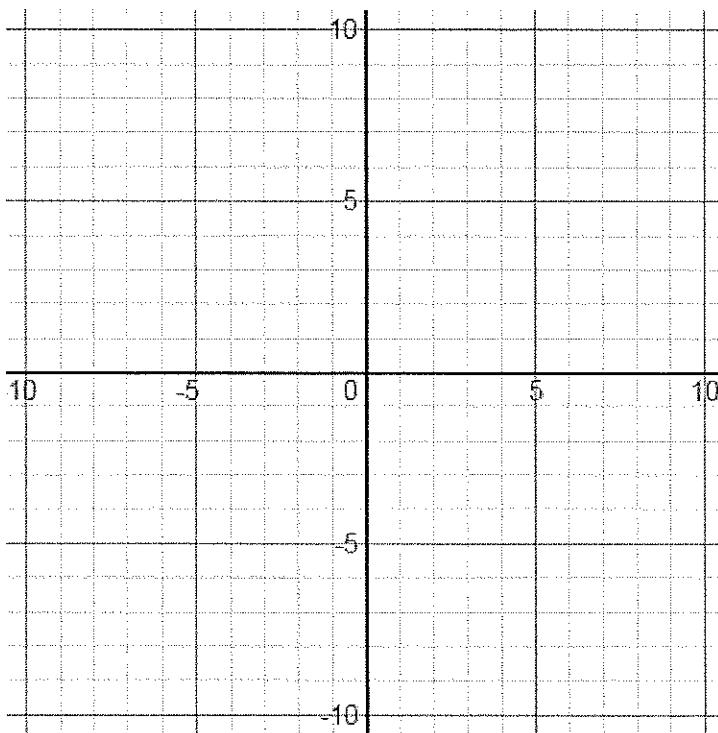
Hole(s)	Vertical Asymptote(s)	Horizontal Asymptote	X-intercept(s)	Y-intercept	Domain	Range

2) Let  $r(x) = \frac{x^2 - 2x - 8}{x^2 - x - 12} = \frac{(x-4)(x+2)}{(x-4)(x+3)} =$   



Hole(s)	Vertical Asymptote(s)	Horizontal Asymptote	X-intercept(s)	Y-intercept	Domain	Range

3) Let  $r(x) = \frac{x^2 - x - 12}{x - 4} = \frac{(x - 4)(x + 3)}{(x - 4)} =$



Hole(s)	Vertical Asymptote(s)	Horizontal Asymptote	X-intercept(s)	Y-intercept	Domain	Range

## DON'T FORGET RADICAL FUNCTIONS...

R1: I can graph square root functions.

R2: I can graph cube root functions.

R3: I can solve radical functions.

Graph the following and identify the domain and range for each:

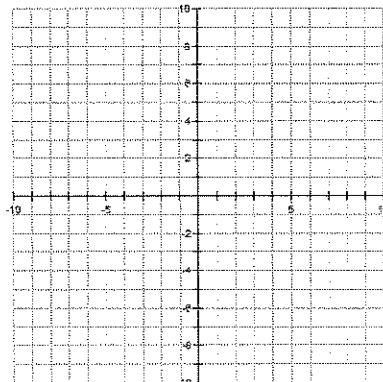
1.  $f(x) = \sqrt{x - 3} - 2$

x	y
1	
2	
3	
4	
7	
10	

Pivot Point: \_\_\_\_\_

Domain: \_\_\_\_\_

Range: \_\_\_\_\_



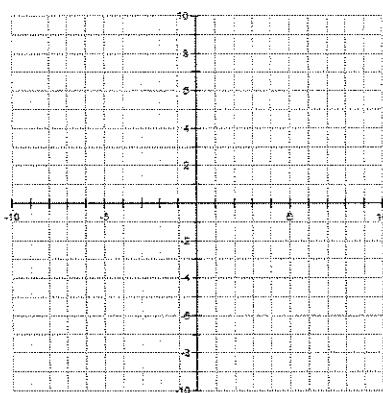
2.  $f(x) = \sqrt[3]{x + 2} - 4$

x	y
-10	
-6	
-3	
-2	
-1	
3	
6	
10	

Pivot Point: \_\_\_\_\_

Domain: \_\_\_\_\_

Range: \_\_\_\_\_



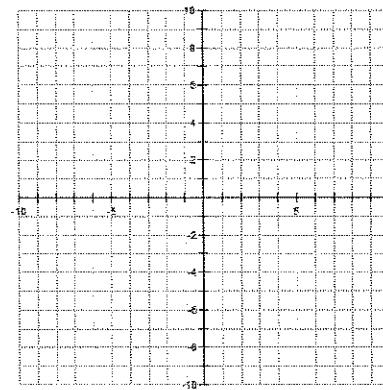
3.  $f(x) = \sqrt[3]{x - 2}$

x	y
-10	
-6	
-2	
1	
2	
3	
6	
10	

Pivot Point: \_\_\_\_\_

Domain: \_\_\_\_\_

Range: \_\_\_\_\_



Identify the domain and range algebraically:

4.  $f(x) = \sqrt{x - 18} + 4$

Domain: \_\_\_\_\_

Range: \_\_\_\_\_

5.  $f(x) = \sqrt[3]{x - 8} + 7$

Domain: \_\_\_\_\_

Range: \_\_\_\_\_

6.  $f(x) = \sqrt{x + 1} - 7$

Domain: \_\_\_\_\_

Range: \_\_\_\_\_

**Solve each of the following equations. Show organized work to support your answers.  
Be sure to check for extraneous solutions.**

$$7. \sqrt{v+7} - 3 = 3$$

$$8. \sqrt{4x} = \sqrt{6x-2}$$

$$9. \sqrt{p+10} = 2$$

$$10. \sqrt[3]{4x+1} = \sqrt[3]{17}$$

$$11. 3\sqrt{x+4} + 13 = 1$$

$$12. 5\sqrt{3x+3} - 9 = 16$$

## Lesson 4-4: Simplifying, Multiplying, and Dividing Rational Expressions

Learning Target: R5: \_\_\_\_\_  
\_\_\_\_\_

Basic Simplifying:

$$1. \frac{x^2+x-20}{x^2-25}$$

$$2. \frac{x^2-36}{x^2-3x-18}$$

$$3. \frac{x^2+2x-15}{x^2-9}$$

$$4. \frac{4x+36}{x^2+13x+36}$$

You Try:

$$5. \frac{x^2-15x+54}{x^2-81}$$

$$6. \frac{x^2-2x-24}{x^2+10x+24}$$

Multiplying and Dividing:

$$1. \frac{x+2}{x^2-4x-12} \cdot \frac{x^2-36}{x-2}$$

$$2. \frac{x^2+x-2}{x^2+5x-6} \cdot \frac{x+6}{x+5}$$

$$3. \frac{1}{3m+6} \cdot \frac{3}{m+3}$$

$$4. \frac{2a+4}{8a^2} \cdot \frac{12a}{a+2}$$

$$5. \frac{y^2-2y-15}{y^2-3y-10} \cdot \frac{y^2-4y+3}{y^2-9}$$

$$6. \frac{2x^2-3x-2}{3x-6} \cdot \frac{6x}{4x^2-1}$$

$$7. \frac{x^2+3x-10}{x^2-2x-15} \div \frac{x^2+x-6}{x^2+6x+9}$$

$$8. \frac{x+5}{2x} \div \frac{x+5}{8}$$

$$9. \frac{m^2}{m+5} \div \frac{m^2+5m}{m^2+10m+25}$$

$$10. \frac{p^2+2p-3}{p^2+2p-8} \div \frac{p^2-1}{p-2}$$

$$11. \frac{\frac{x^2 - 4x}{x^2 - 8x + 16}}{\frac{12}{2x - 8}}$$

$$12. \frac{\frac{b+3}{b^2 + 6b + 9}}{\frac{b+2}{b^2 - 9}}$$

You Try:

$$13. \frac{x-1}{x+5} \cdot \frac{x^2+x-20}{x^2-8x+7}$$

$$14. \frac{x+9}{7x^3} \cdot \frac{14x}{4x+36}$$

$$15. \frac{x^2+1x-30}{x-7} \div \frac{x^2+12x+36}{7-x}$$

$$16. \frac{x-2}{(4x+3)^2} \div \frac{(x-2)^2}{4x+3}$$

## Lesson 4-5: Adding and Subtracting Rational Expressions

Learning Target: R6: \_\_\_\_\_

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$$1. \frac{5y}{4y^2} + \frac{12}{4y^2} + \frac{3y}{4y^2}$$

$$2. \frac{9a^3}{3a^2} - \frac{6a}{3a^2}$$

$$3. \frac{2t}{3t-12} - \frac{8}{3t-12}$$

$$4. \frac{w^2}{w^2-9} + \frac{2w-15}{w^2-9}$$

$$5. \frac{a^2+10}{a^2-4} - \frac{7a}{a^2-4}$$

$$6. \frac{y^2-13}{y^2-25} + \frac{3(1-y)}{y^2-25}$$

$$7. \frac{2}{y^2} + \frac{1}{3} - \frac{5}{6y^2}$$

$$8. \frac{x^2-12}{x^2-4} + \frac{2}{x-2}$$

$$9. \frac{d}{d^2-1} - \frac{d}{d-1}$$

$$10. \frac{x}{x+1} + \frac{8}{x-2}$$

$$11. \frac{3}{m+2} + \frac{m^2}{m^2-4} - \frac{1}{m-2}$$

$$12. \frac{6}{a^2-2a-35} - \frac{2}{a^2+9a+20}$$

**You Try:**

$$13. \frac{3}{x-1} + \frac{5}{x+1}$$

$$14. \frac{x}{x^2-2x-3} + \frac{x}{x^2+5x+4}$$

$$15. \frac{2x+1}{x-4} - \frac{x+5}{x-4}$$

$$16. \frac{6}{5x+15} - \frac{1}{x+3}$$

## Lesson 4-6: Solving Rational Expressions

Learning Target: R7: \_\_\_\_\_  
\_\_\_\_\_

$$1. \frac{4}{x^2} = \frac{1}{9}$$

$$2. \frac{2t}{t-2} = \frac{t+4}{t-2}$$

$$3. \frac{b+2}{b-3} = \frac{3b-4}{b-3}$$

$$4. \frac{w^2}{w-4} - \frac{8}{w-4} = \frac{2w}{w-4}$$

$$5. \frac{5}{x-4} = \frac{3}{x}$$

$$6. \frac{p^2}{p+2} = \frac{4p+12}{p+2}$$

$$7. \frac{4a^2-9}{2a-3} = 9$$

$$8. \frac{x^2-3x+4}{x-4} = -4$$

$$9. \frac{2}{3x^2} = \frac{1}{x} - \frac{1}{3}$$

$$10. \frac{x}{x-4} = \frac{x+10}{x-2}$$

$$11. \frac{p-1}{p+3} - \frac{2}{p-3} = \frac{7-3p}{p^2-9}$$

$$12. \frac{x}{x-2} + \frac{2}{x+3} = \frac{3x+4}{x^2+x-6}$$

**You Try:**

$$13. \frac{1}{6x^2} + \frac{1}{6x} = \frac{1}{x^2}$$

$$14. \frac{x-2}{x-5} = \frac{3}{x-5}$$

$$15. x + \frac{7}{x} = -8$$

$$16. \frac{6x}{3-x} = \frac{2x^2}{x-3}$$

**UNIT 4 TEST REVIEW**

Name: \_\_\_\_\_

**Graphing Rational and Radical Functions and their characteristics:**

$$1. \ y = \frac{x+5}{x^2+2x-15} = \frac{1(x+5)}{(x-3)(x+5)}$$

- A. Holes:
- B. Vertical Asymptote(s):
- C. Horizontal Asymptote(s):
- D. y-intercept:

$$2. \ y = \frac{x^2+3x-18}{x^2+2x-15} = \frac{(x-3)(x+6)}{(x-3)(x+5)}$$

- A. Holes:
- B. Vertical Asymptote(s):
- C. Horizontal Asymptote(s):
- D. y-intercept:
- E. x-intercept(s):

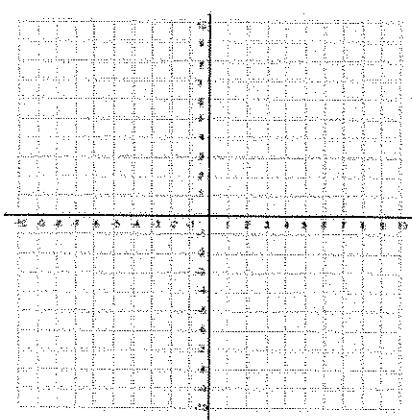
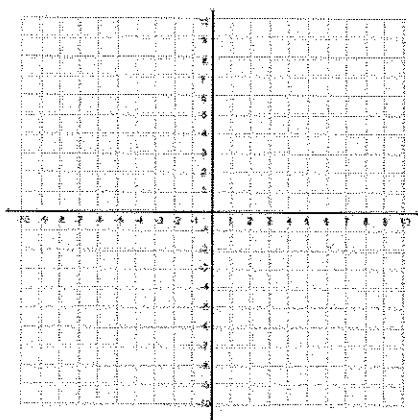
**Use the Graphs to the right to graph the functions if you need to:**

3.  $f(x) = \sqrt{x+5} + 2$

4.  $f(x) = \sqrt[3]{x+7} - 5$

- A. Domain:
- B. Range:

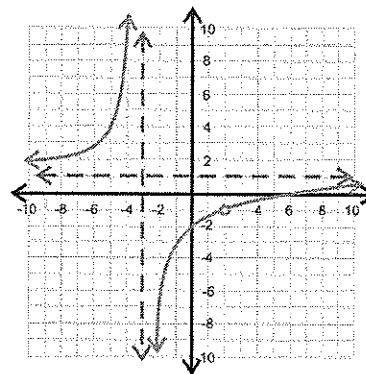
- A. Domain:
- B. Range:



5. Use the graph to the right to identify the following:

A. Domain:

B. Range:



6. Simplify the following:

A.  $\frac{x^2 - 13x + 30}{x^2 - x - 6}$

B.  $\frac{x^2 - 9}{x + 3} \cdot \frac{x^2 + 4x + 3}{2x - 6}$

C.  $\frac{x+2}{x+5} \div \frac{x^2 + 6x + 8}{x+5}$

D.  $\frac{x}{x-4} - \frac{x+5}{x-4}$

E.  $\frac{x}{x^2 - 3x - 10} + \frac{3}{x-5}$

7. Solve the following:

A.  $7\sqrt{3x+4} - 5 = 30$

B.  $\sqrt[3]{3x+5} + 10 = 12$

C.  $\sqrt{8x+2} = \sqrt{2x+20}$

D.  $x + \frac{12}{x} = -7$

E.  $\frac{2}{x+3} = \frac{1}{x-4}$

## **UNIT 4 – Graphing Radical and Rational Functions**

### **UNIT 4 Reflection**

Name: \_\_\_\_\_

What about this unit did you find to be the easiest? \_\_\_\_\_

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What about this unit did you find to be the most difficult? \_\_\_\_\_

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Are you on track to pass this course? If not, what is your game plan? What are you going to do to help yourself master this course?

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What are your plans after high school? Do you plan to go to college? Go into the work force, etc? Do you feel prepared? Have you taken the SAT/ACT or are you scheduled to take it?

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Where do you see yourself in the next 10 years? What are your goals?

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