NAME: _____ PERIOD: _____

UNIT 6

CIRCLES PACKET

Standard Form and General Form

Standards:

MGSE9-12.G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

Learning Targets:

1. I can determine the center and the radius of a circle when given an equation in standard form.
2. I can write the equation of a circle when given the center and radius of a circle.
3. I can convert the equation of a circle in general form to standard form.
4. I can determine the center and the radius of a circle when given an equation in general form.
5. I can use the equation of a circle to solve real world problems.

Circles – Standard Form

A Circle is the set of all points (x, y) that are equidistant from a fixed point called the center of the circle.

The distance "r" between the center and any point (x, y) on the circle is the radius.

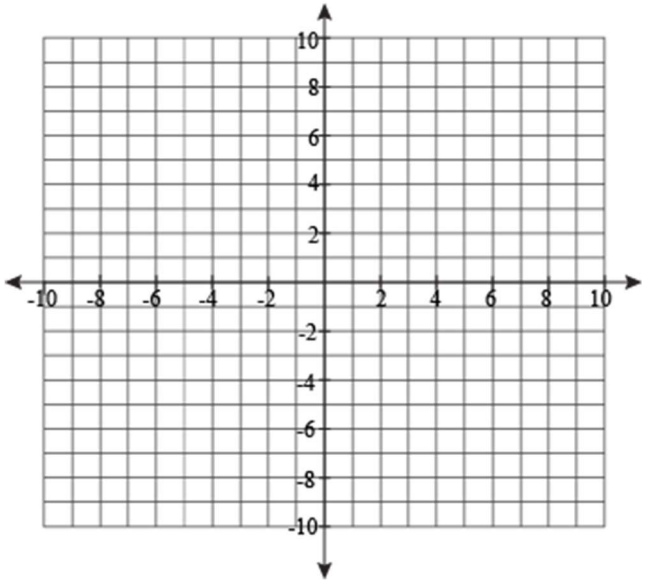
Standard Form: $\mathbf{(x - h)^2 + (y - k)^2 = r^2}$

Center (h, k)
Opposite Signs

radius is the square root of r^2

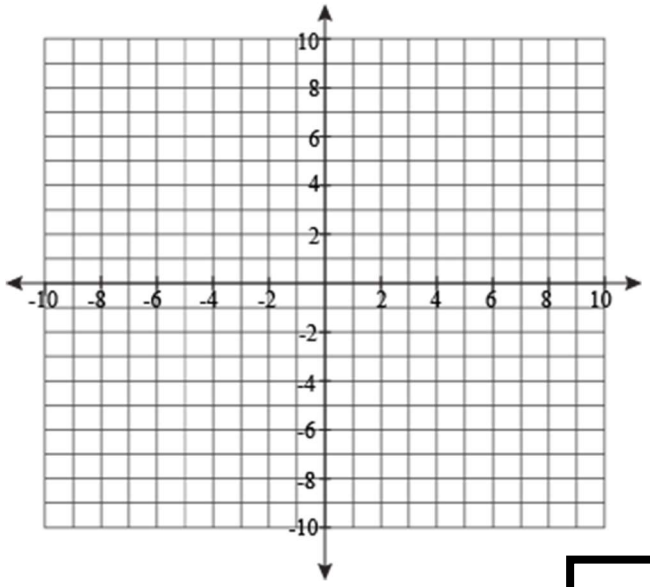
Ex 1: $(x - 6)^2 + (y - 2)^2 = 4$

- Identify the Center _____
- Find the Radius _____
- Graph the circle
  - Plot the Center
  - Plot 4 points that are a radius away from the center.
  - Draw a circle through the points



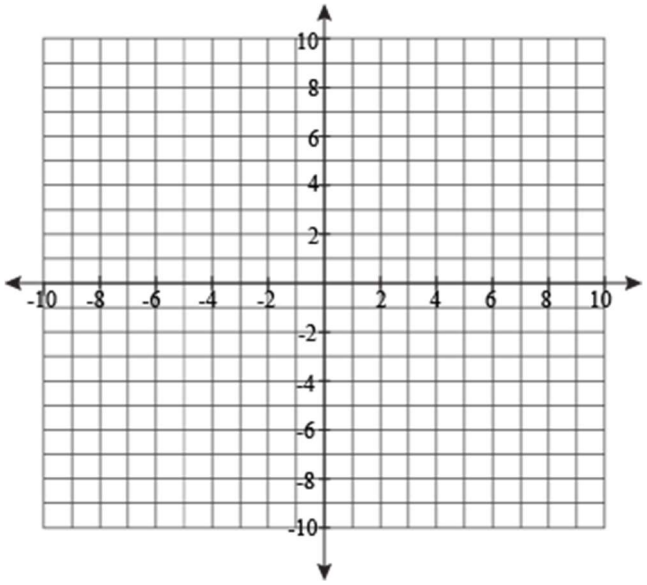
Ex 2: $(x + 6)^2 + (y + 2)^2 = 9$

- Identify the Center _____
- Find the Radius _____
- Graph the circle
  - Plot the Center
  - Plot 4 points that are a radius away from the center.
  - Draw a circle through the points



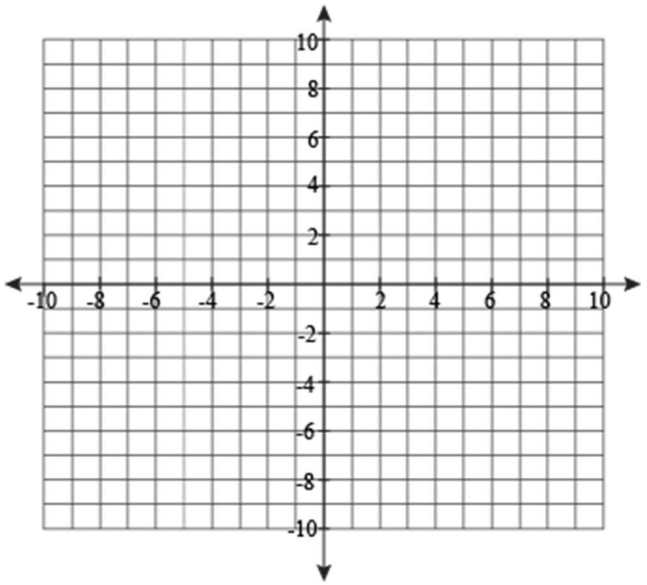
Ex 3: $x^2 + (y + 3)^2 = 25$

- Identify the Center _____
- Find the Radius _____
- Graph the circle
  - Plot the Center
  - Plot 4 points that are a radius away from the center.
  - Draw a circle through the points



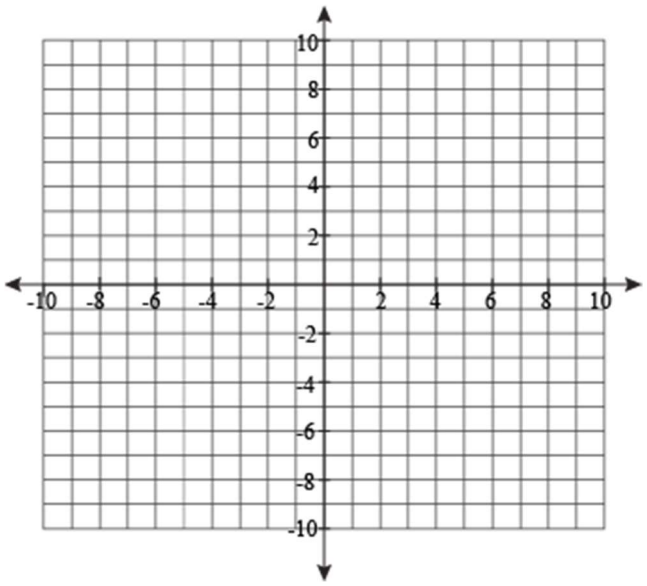
Ex 4: $(x - 4)^2 + y^2 = 9$

- Identify the Center _____
- Find the Radius _____
- Graph the circle
  - Plot the Center
  - Plot 4 points that are a radius away from the center.
  - Draw a circle through the points



Ex 5: $x^2 + y^2 = 36$

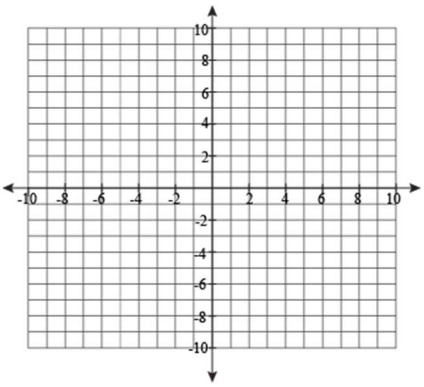
- Identify the Center _____
- Find the Radius _____
- Graph the circle
  - Plot the Center
  - Plot 4 points that are a radius away from the center.
  - Draw a circle through the points



YOU TRY:

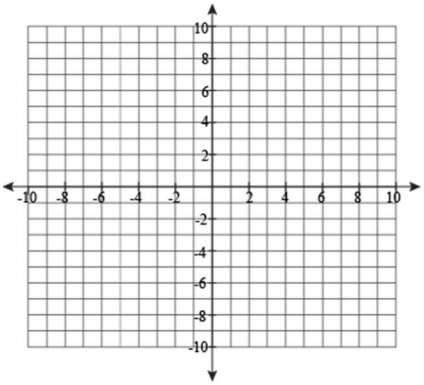
1. $(x - 2)^2 + (y + 8)^2 = 4$

Center _____
Radius _____



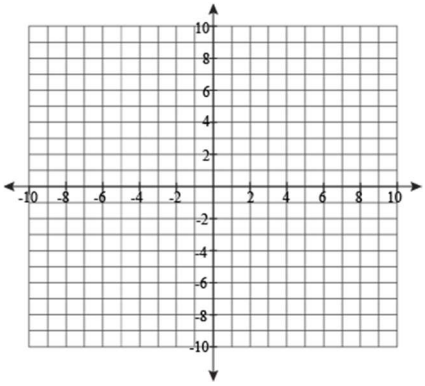
2. $(x + 7)^2 + (y + 2)^2 = 1$

Center _____
Radius _____



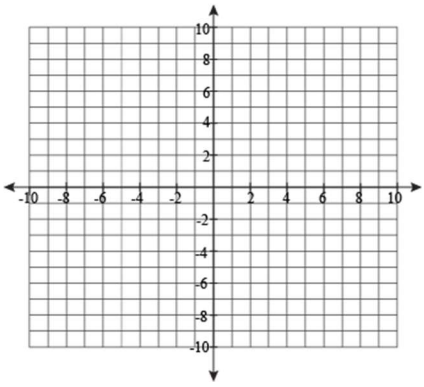
3. $(x + 1)^2 + (y - 7)^2 = 9$

Center _____
Radius _____



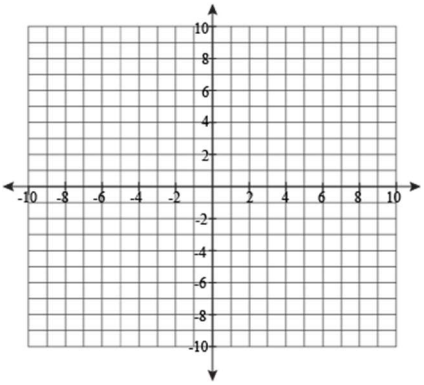
4. $(x - 3)^2 + (y - 1)^2 = 81$

Center _____
Radius _____



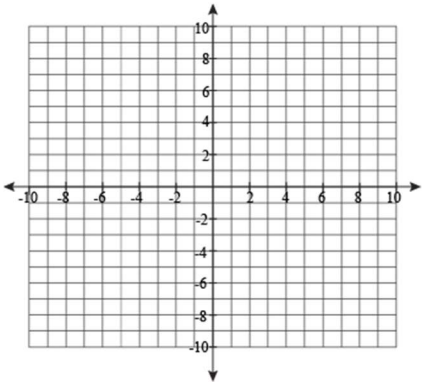
5. $x^2 + y^2 = 4$

Center _____
Radius _____



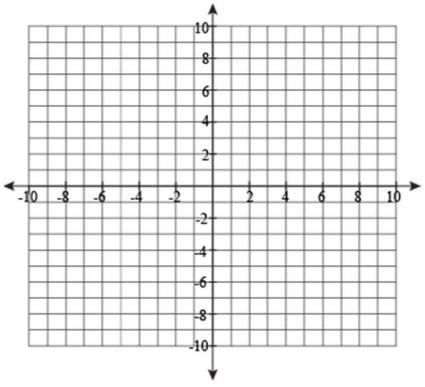
6. $(x - 2)^2 + y^2 = 64$

Center _____
Radius _____



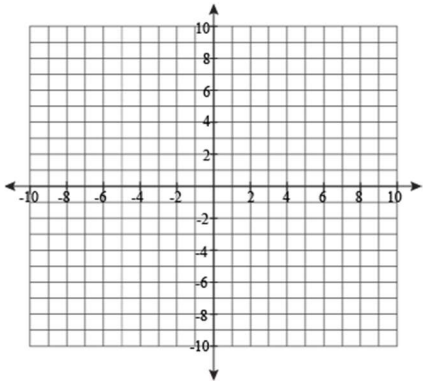
7. $x^2 + (y + 7)^2 = 9$

Center _____
Radius _____



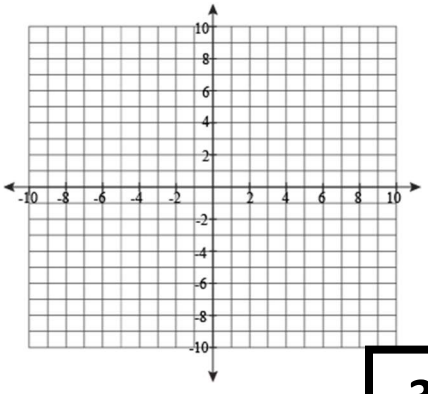
8. $x^2 + y^2 = 49$

Center _____
Radius _____



9. $(x + 4)^2 + (y - 5)^2 = 16$

Center _____
Radius _____



Writing the Equation of a Circle: $(x - h)^2 + (y - k)^2 = r^2$

Given the following Center and radius, write the equation of the circle in standard form.

Ex 1: Center (9, 3)
Radius = 4

Ex 2: Center (-4, 2)
Radius = 3

Ex 3: Center (5, -6)
Radius = 5

Ex 4: Center (0, 4)
Radius = 7

Ex 5: Center (0, 0)
Radius = 9

Ex 6: Center (-8, 0)
Radius = 11

You Try:

1. Center (-4, 10)
   Radius = 6

2. Center (15, -3)
   Radius = 7

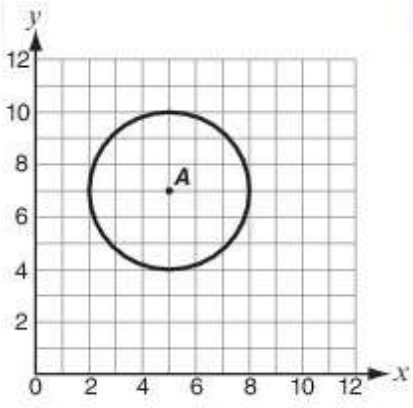
3. Center (-3, 7)
   Radius = 10

4. Center (-5, -10)
   Radius = 8

5. Center (0, 0)
   Radius = 9

6. Center (0, 5)
   Radius = 5

The following graph represents a cell phone tower and it's range.



7. Write the equation of the circle:

   Center:

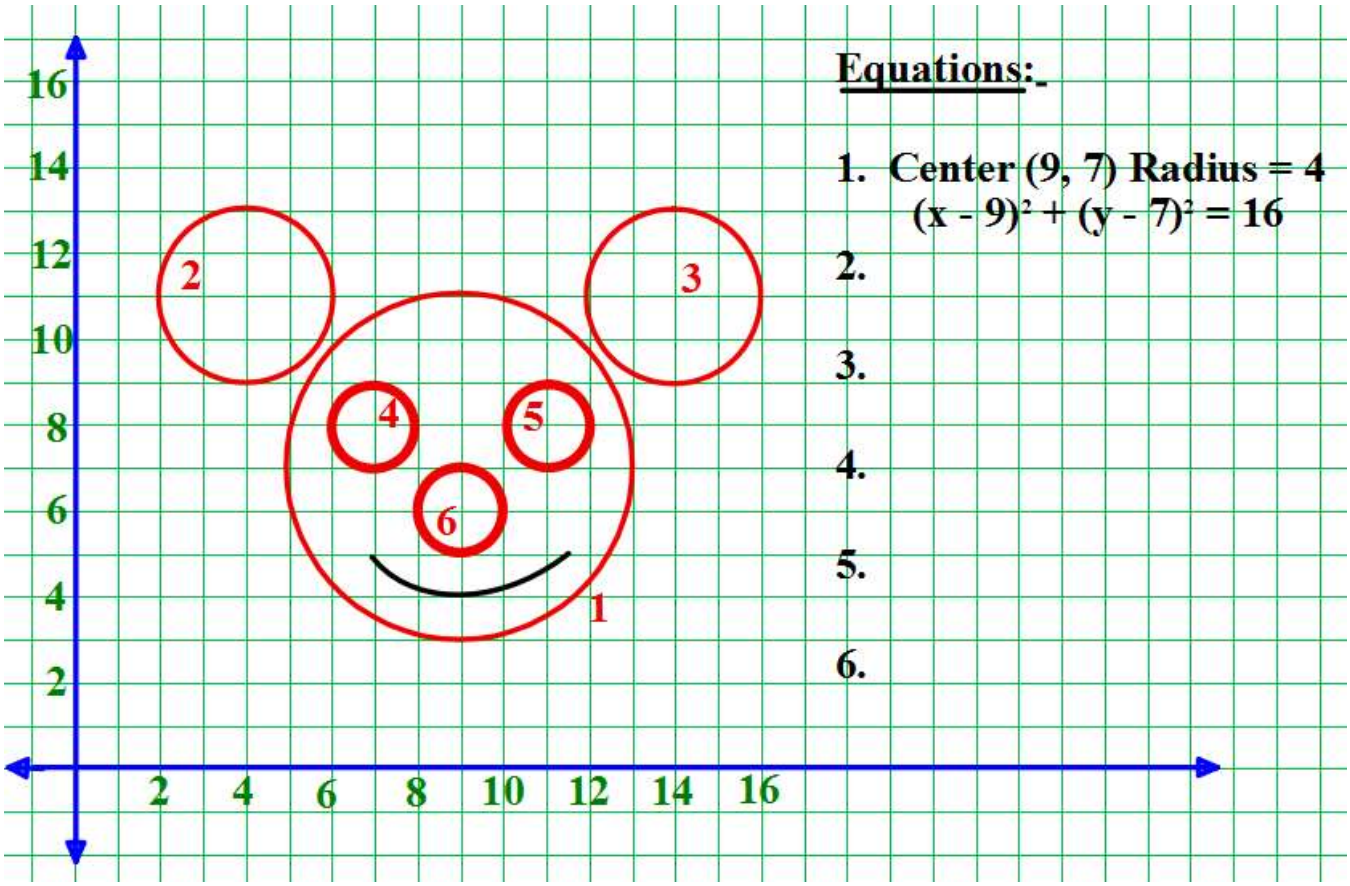
   Radius:

   Equation:

8. If your house was located at the coordinate (8, 10) would you have cell service?

9. If your house was located at the coordinate (4, 6) would you be on the interior or exterior of the circle?

Using the following picture, identify the center and radius for each circle and write the equation of each circle.



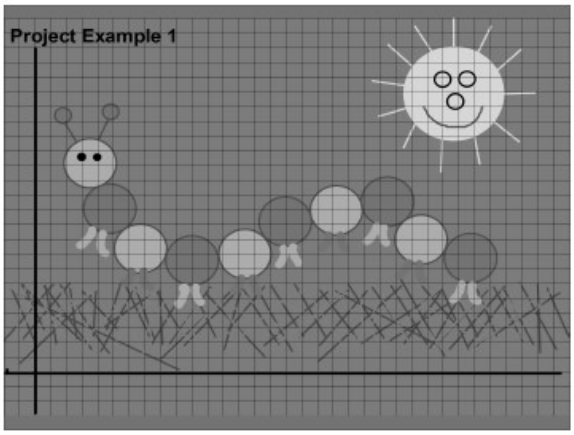
PROJECT

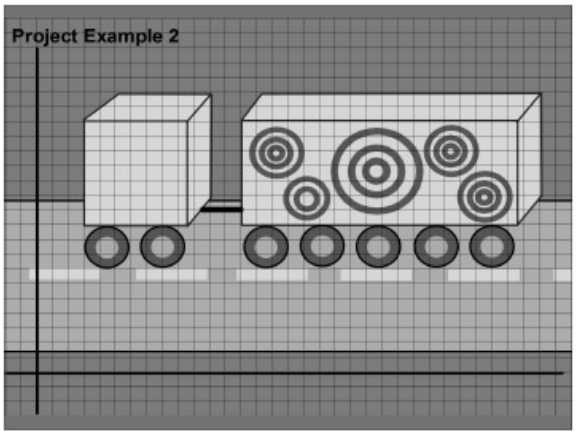
You will be creating a picture / design circles. You must have at least 10 circles. For every additional circle you add (up to 10) you will get 1 bonus point (max of 10 bonus points) It should be evident that you took time to create your picture. You shouldn't just have random circles. Your circles should create an actual picture or a pattern. Your design can include other things as well to make the picture work. See examples. You should be creative, use color, and be extremely neat. Circles should be drawn with a compass or by tracing circular objects and should not be freehanded.

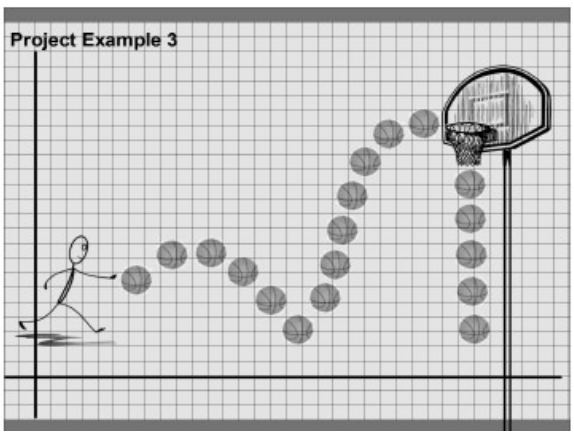
This is a test grade and you can earn up to 110 points if you add bonus circles.

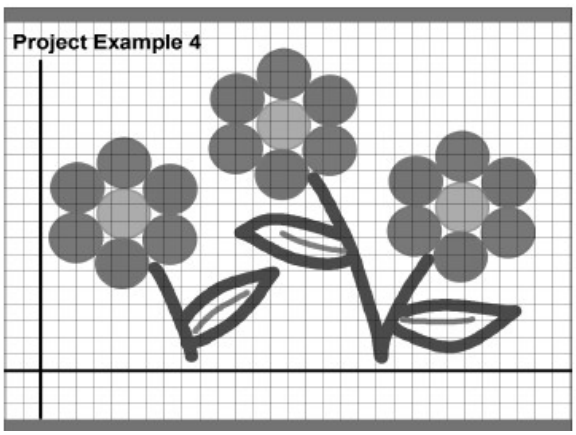
	16 – 20 points	11 - 15 points	6 to 10 points	0 to 5 points
Required Components	At least 10 circles are included in your design.	At least 8 circles are included in your design.	At least 6 circles are included in your design.	Less than 6 circles are included in your design.
Creativity	There is a clear picture displayed (circles are not randomly placed) Picture must be detailed, creative, and appropriately colored. Creativity exceeds expectations.	There is a clear picture displayed. Picture is creative and detailed. Picture is colored but creativity is not "outside" the box.	There is a clear picture displayed but picture is not very creative or is somewhat detailed or picture is not colored.	There is not a clear picture displayed or picture lacks creativity and lacks detail.
Neatness	Circles are clearly drawn with a compass or appropriately traced. Coloring is neatly done and not just scribbled on.	Drawings are drawn with appropriate tools looks somewhat messy.	Circles are not appropriately drawn. Picture is somewhat neat.	Circles are not appropriately drawn and coloring is not neat. Picture appears sloppy.
Equations / Labeling	All circles are labeled with the appropriate equation. Number each circle based on what you put on your chart. Equations are 100% accurate based on drawings and calculations.	All circles are labeled but there are some issues with accuracy in the equations or at least 8 of the equations are labeled appropriately with 100% accuracy.	Circles are labeled but there are some issues in accuracy.	Circles are not labeled with appropriate equations or there are some extreme issues with the accuracy of equations
Paperwork / Calculations	For all circles, you must identify the equation, the center, and the radius.	Equations are accurately shown but are missing 1 of the required components.	Equations are shown but there are some accuracy issues or 2 of the required components are missing.	Equations are shown but there are some major accuracy issues or more than 2 of the required components are missing.

Some Ideas… but these are not to be copied. Be original.



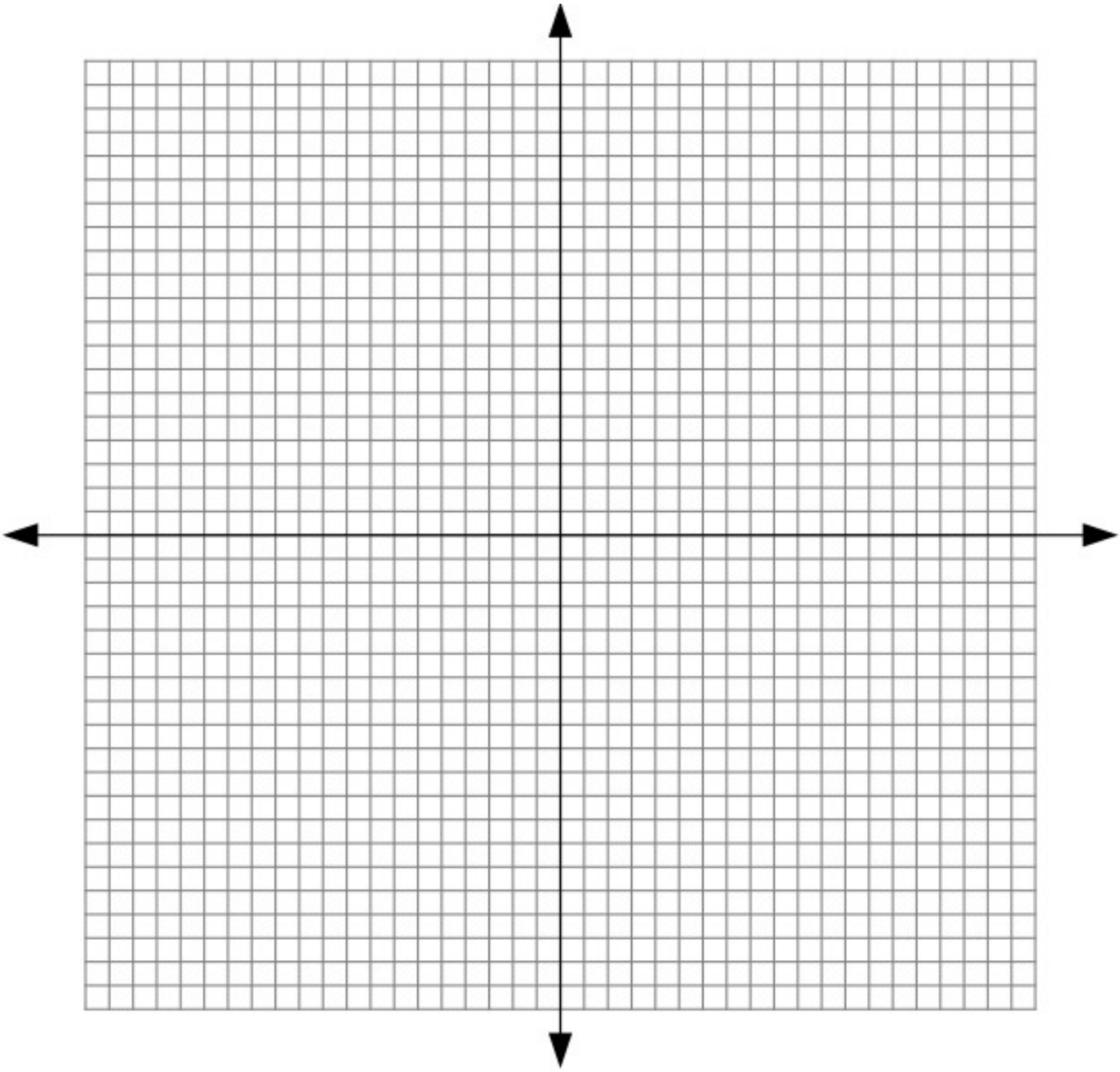






Do a <u>rough draft</u> below…

Final Project should be done on a separate project sheet… this is only a rough draft and is not part of your project grade. This does not need to be colored or be very neat or be drawn with a compass. You do not have to tell me the equations here either. This is just a place for you to get your ideas down on paper. My suggestion… Make sure your circles have whole number centers and radii. This will make coming up the equations later much easier.



Circles – General Form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

A and C must be equal and have the same sign in order to be a circle. Other characteristics in this same formula form the equations of an ellipse, hyperbola, or parabola.

If you are given an equation in this form you can convert it to standard form by completing the square.

STEPS TO COMPLETING THE SQUARE:

1. Rearrange the equation putting the x's together and the y's together leaving a blank after each one. Also move the the constant to the other side with two blanks after it.
2. Take ½ of the 2nd x coefficient and square it. Add this number in the blank following the x's. Do the same for the y's. Also… whatever you add to the left side, remember you must add to the right side. This preserves equality.
3. Factor each trinomial on the left side and simplify the right side.

Ex 1. $x^2 + y^2 + 8x - 2y + 13 = 0$

Ex. 2. $x^2 + y^2 + 6x - 10y + 9 = 0$

Ex. 3. $x^2 + y^2 - 4x + 8y + 4 = 0$

Ex. 4. $x^2 + y^2 + 16x - 12y - 21 = 0$

Ex. 5. $x^2 + y^2 - 14\,x + 6y - 6 = 0$

You Try:

Find the center and radius of the circle whose equation is written in general form.

1. $x^2 + y^2 + 4x - 2y - 11 = 0$

2. $x^2 + y^2 - 6x + 14y + 33 = 0$

3. $x^2 + y^2 - 4x + 2y - 4 = 0$

4. $x^2 + y^2 + 2x + 18y + 46 = 0$

5. $x^2 + y^2 + 4x - 16y + 32 = 0$

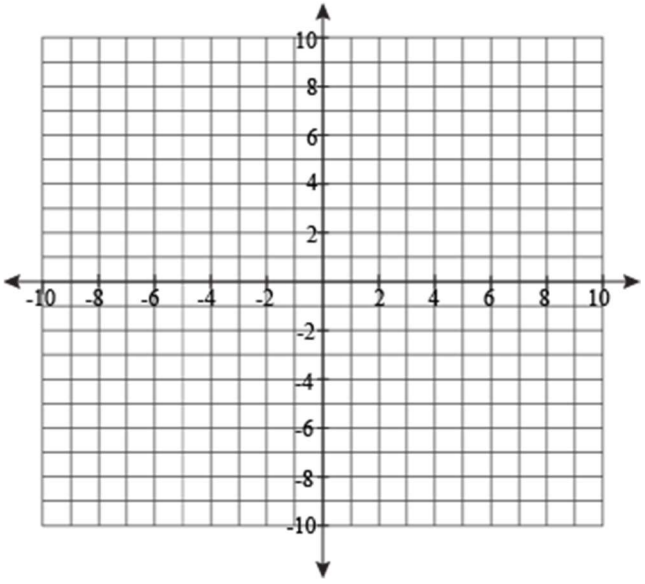
6. $x^2 + y^2 + 4x - 6y - 3 = 0$

Find the center and radius of the circle given the following equation and <u>graph</u> the equation.

7. $(x - 1)^2 + (y + 5)^2 = 9$

Center:

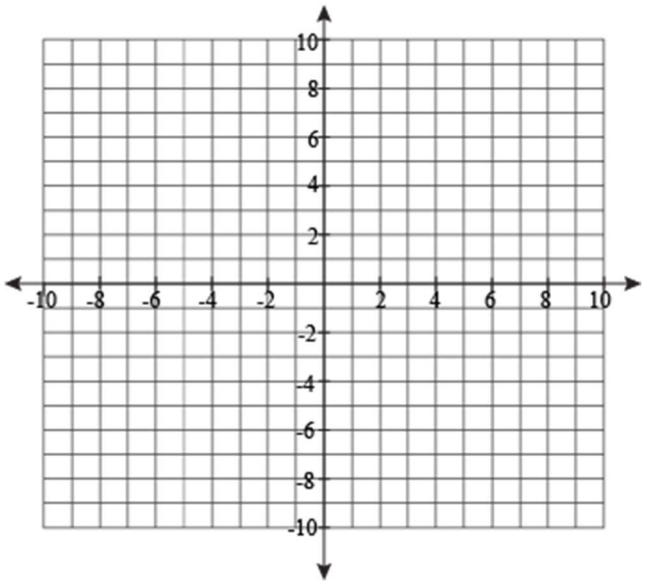
Radius:



8. $(x + 4)^2 + (y + 6)^2 = 4$

Center:

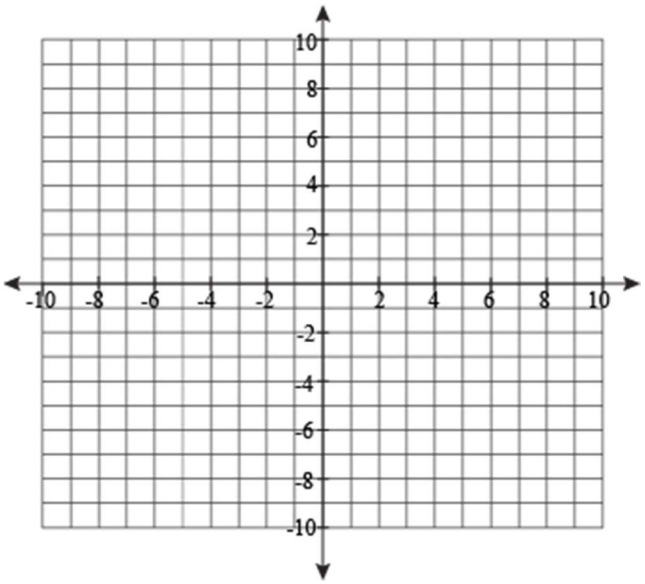
Radius:



9. $x^2 + y^2 = 81$

Center:

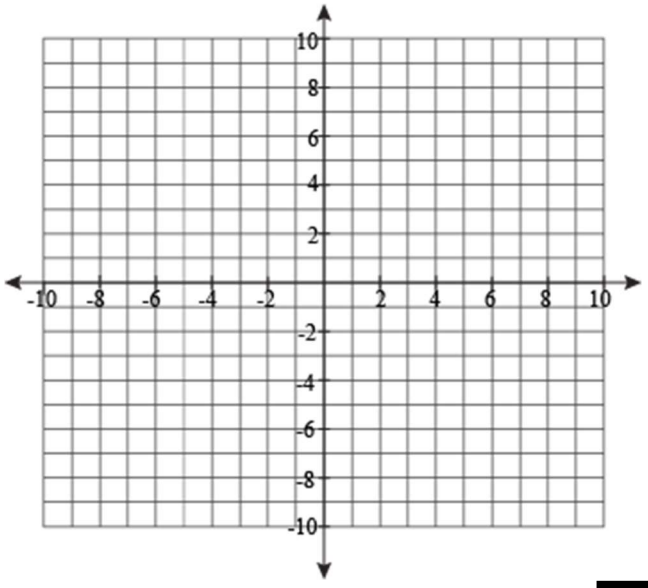
Radius:



10. $x^2 + (y - 5)^2 = 25$

Center:

Radius:



Given the following center and radius, write the standard form of the equation.

11. Center (-8, -5) Radius = 9

12. Center (-4, -9) Radius = 7

13. Center (0, 0) Radius = 5

14. Center (0, 4) Radius = 8

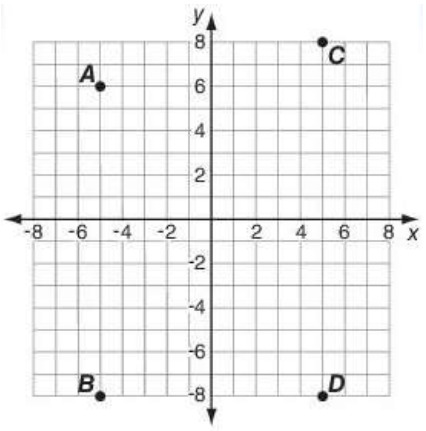
15. Center (-5, 8) Radius = 6

16. Center (12, 11) Radius = 10

17. Center (-4, -8) Diameter = 6

18. Center (-5, 2) Diameter = 16

19. Which point is the center of the circle with equation $(x + 5)^2 + (y + 8)^2 = 36$?



20. The location of cell phone tower A is shown on the coordinate plane as shown. Integers represent miles. The tower is located at (5, 7) and has a transmission range of 3 miles. Write the equation that represents the position and range of tower A?